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COMMONWEALTH BUREAU OF CENSUS AND STATISTICS.
MELBOURNE, AUSTRALIA.

Price-Indexes, their Nature and Limitations,
the Technique of Computing them, and
their Application in Ascertaining the
Purchasing-Power of Money.

Prepared under instructions from the Minister of State
for Home and Territories.

BY

G. H. KNIBBS, C.M.G.,

Honorary Fellow of the Royal Statistical Society, Honorary
Member of the American Statistical Association, and of
the Société de Statistique de Paris, Membre de l'Institut
International de Statistique; etc., etc., etc.

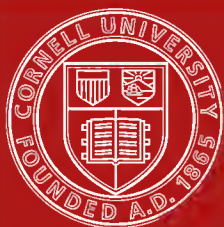
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This Appendix will appear later as a part of the Labour Report No. 9, 1918.
This Report, it is hoped, will be published about May, 1919.

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CORRIGENDA.

Page 4, line 19.—For “beend one” read “been done.”

Page 5, line 12.—After “correct” insert “, however, only.”

„ „ 31.—For “required” read “requires.”

Page 12, line opposite “commodity C.”—For “446 $\frac{17}{8}$ ” read “446 $\frac{3}{4}$,” and on line below in table and second line in next paragraph, for “696 $\frac{17}{8}$ ” read “696 $\frac{3}{4}$.”

Page 16, line 8.—For “1000/1422.0” read “1000/1442.0.”

Page 20, line 11 from bottom of page.—For “or U_c ” read “or U'_c .”

Page 21, line 9.—Insert comma between “are” and “rigorously.”

„ „ 26.—For “divergence” read “divergences.”

Page 23, line 3 from bottom.—For “Geometri” read “Geometric.”

Page 28, line 33.—For “ Q/Q ” read “ Q_n/Q_1 .”

Page 33, line 5.—“Commodity” should be over the words “For P.”

Page 35, line 4 from bottom of page.—For “nce” read “once.”

Page 38, line 6 from bottom of page.—For “ a ” read “ a' .”

Page 42, line 13.—For “Chapter” read “Part.”

Page 43, line 27.—For “ b ” read “ b' .”

Page 44, in table, last column shewing price-levels, omit “%”

„ in second table, last column shewing price-levels.—For “1543.1” read “1542.9” and for “1078.9” read “1078.7.”

Page 47, line 3.—For “*in intenso*” read “*in extenso*.”

Page 48, note to second table.—Insert full stop after “comparison.”

APPENDIX I.

PART I.

INTRODUCTORY REMARKS ON PRICE-INDEXES.

SYNOPSIS.

1. The significance of price-indexes.
 2. Necessity of accurate conceptions regarding the nature of a price-index.
 3. Aim of the present article.
 4. Complexities may be avoided.
 5. Difference between price-ratios and price-indexes.
 6. Popular view of relations between price-ratios and price-indexes.
 7. Purpose of price-indexes must be considered.
 8. Exposition of nature of technique.
 9. Differences between household-budgets and composite-units.
-

1. **The significance of price-indexes.**—Changes in the price of commodities have, of course, always been of interest to students of economics. They mark the variation of the relation between one commodity, viz., gold, and all other commodities which have an exchange value, and consequently indirectly shew the exchange relationships between them all. Fluctuations of price, therefore, have always been of considerable academic interest. Owing to certain recent economic tendencies, these fluctuations have become of still greater moment, owing to the increased importance of certain questions, among which may be mentioned that concerning the payment for services rendered by wages of equivalent purchasing-power, i.e., by a rate of wage which has regard to the commodity-value thereof. Owing to this, the necessity of accurately ascertaining the purchasing-power of money (and therefore of wages) has been accentuated, and in the same connection so also have various other questions concerning the system of exchange-value relations between money and commodities generally. If we use the expression "*the purchasing-power of money*" to denote some general relation between the unit of currency—say £1 sterling in English communities—and the satisfaction, both as to commodities and services required, of human needs, it is obvious that a clear understanding of what is really meant by this term is a matter of no small moment.

A means, widely adopted, for the purpose of measuring variations in the cost of commodities and services is to use *price-indexes*. A "price-index" is the reciprocal of an index of purchasing-power and, as explained hereinafter, shews the increase or decrease at different periods in the cost of a definite group of commodities and services. An index of purchasing-power on the other hand would shew the variation in the purchasing-power of any particular money-unit.

How we are to measure or estimate these, and in what way can a "price-index" disclose the variations of purchasing-power are matters of common concern. "How can price-indexes be accurately calculated?" ; "What are the limitations affecting any attempt to determine them or purchasing-power accurately?" and "How should price-indexes be used for equalising wages, so as to make their commodity and service-purchasing efficiency constant?"—if so desired or if possible—are self-evidently matters which must engage the attention of all interested in the trend of human affairs.

Whether rightly or wrongly, a dictum which at the present time is somewhat insistently asserted, is that wages should be estimated, not by mere expression in the form of currency, but by the quantity of any desired commodities and services which they will purchase.

As is shewn by the large number of treatises dealing with questions of this nature, the problem is not quite so simple and obvious as it might at first sight appear.

2. Necessity of accurate conceptions regarding the nature of a price-index.—The truth of this statement is evidenced by the doctrines regarding price-indexes to be found in certain economic treatises, and also by the fact that price-indexes are not unfrequently so ill-founded as to be quite misleading. For this reason an exposition of the whole matter of a somewhat brief and technical nature (at least in so far as the appendix was concerned), was given by me in Labour Report No. 1 of the Labour and Industrial Branch of the Commonwealth Bureau of Census and Statistics, published 5th December, 1912, pp. 1-96, with appendices, pp. i. to lxii. Experience has shewn, however, that this exposition has often been misunderstood and misquoted. This, and the fact that the algebraical treatment has sometimes been misdescribed and misapplied, have disclosed the desirability of setting out the whole of the matter, in a more elementary form, and this has been one in the following pages.

Those who desire to know the detail of various methods that have been used for obtaining price-indexes are referred to the Report above mentioned, and to the bibliography therein contained, particularly such works as Professor Irving Fisher's "Purchasing Power of Money" (New York, 1911), and the "Measurement of General Value," by C. M. Walsh (New York, 1901).

3. Aim of the present article.—An endeavour has been made here, to set forth the essentials of the problem and of its solution, with sufficient illustration to enable anyone, who, being qualified to form a competent opinion, will give the matter serious attention. Any attentive student can thoroughly understand the question at issue; can realise what is ideally required, and can appreciate the limitations of the practical situation in regard to ascertaining price-indexes for different localities at one period of time, and for the same locality at different periods. It ought to be added, however, that the matter is not likely to be understood without close attention, and that an erroneous point of view may make its apprehension by no means easy.

4. Complexities may be avoided.—It is shewn herein that certain complexities introduced into the subject by various political economists, have only beclouded what is really a very clear and definite issue. It happens that the simple and very elementary notion of the "man in the street" as to what is necessary is, in this case, correct, viz., that an unequivocal determination must be based upon the series of commodities used, the quantities being so taken as to be *proportionate to actual usage*. Following the suggestion of Professor Irving Fisher, I have called such a series a "composite-unit," and it is on the cost of this unit or the aggregate expenditure thereon at different times, or at different places, that a definite idea can be formed of the changing value of money in relation to commodities. It may be mentioned that the British Board of Trade and the American Bureau of Labour and Statistics have, since the publication of the original report, adopted the method which I have called herein "the method of aggregate expenditure," that is, the measurement of the changing value of currency by comparisons of the cost of the composite-unit.*

* Mr. Wesley C. Mitchell, in his article in the "Bulletin of the U.S. Bureau of Labour Statistics," No. 173, July 1915, p. 35, says:—

"Still later (1912) the method practiced by Dun was adopted by the Commonwealth Statistician of Australia as the basis of his official series. However, after he had calculated the aggregate expenditure of Australians upon this bill of goods in terms of pounds sterling, he threw these pecuniary sums back into the form of relative numbers on the scale of 1000."

Dun's review referred to was dated 1901. Dun's method was certainly right in principle, in that the price was multiplied by the quantity supposed to be consumed in the course of a year by an average individual.

No exposition appears to have been given by Dun of the justification of this method, nor was its identity with the price-ratio method, when geometric means of the relative expenditures are made the basis of combination, shewn. But this was demanded by the then existing state of the theory of price-indexes, and the aggregate-cost method was adopted by me only after establishing the fact that *its merits are unique*, inasmuch as it is not only perfectly definite in its significance, but also gives, in the most simple way, a result always identical in practical cases with that properly deduced from price-ratios, viz., by taking the weighted geometric means on a mean commodity-basis for the years compared, the weights also being the means of the relative expenditures on the commodities for the compared years.

The remarks on p. 101 of the Labor Bulletin (above quoted) regarding Dun's and Gibson's index-numbers seem to imply that the weighting-method is somewhat loose, and it would appear also to be based upon value-ratios.

5. **Difference between price-ratios and price-indexes.**—It is convenient to call the ratio of the price of any individual commodity at one time, to the price at any other time, the *price-ratio* of the latter date as compared with the former, and to reserve the term "*price-index*" for the price-ratio for the same type of comparison applied not to single commodities, but to special groups of commodities, the individual items of the group being combined in some definite way. If, taking all facts of usage into account, the prices of all commodities were in the same ratio, this ratio would also be the price-index for the whole, and the items could in this case be grouped in any way whatsoever, that is, the price-index would be independent of the relative quantities of each. This perhaps throws some light upon the *popular opinion that the price-ratio or percentage of rise and fall in price is the main fact with which we have to deal*, a view which is correct if all qualifying circumstances are fully taken into account.

When the price-ratios are different for different commodities, it is self-evident that the extent of their usage has a profound influence on the result, for example, changes in the price of caviare and champagne are practically of no general importance compared with the change in the prices of bread, meat, butter, milk, etc., for the reason (statistically) that the aggregate quantities used are relatively negligible and thus have no sensible influence upon the general result. We may say, therefore, that relatively they have no *weight*. In proportion as the relative usage is considerable, so must the "weight" increase which would be attributed. Thus a system of weights must be applied to the price-ratios of the various commodities, and their effect thus taken into account in any general determination.

6. **Popular view of relations between price-ratios and price-indexes.**—In a loose kind of way it is popularly felt that *the relative importance of commodities must vary with the proportion of the expenditure thereon*. From one point of view, this is true. In so far, however, as the commodities are necessities of life, it may be that the actual quantities used determine their relative importance, or yet again, that both elements, viz., expenditure and quantity, must sometimes be taken into account. In regard to this latter observation it may also be noticed that in all cases where the amount available for the purchase of what one required is limited, the variations of expenditure may involve re-adjustments among the quantities of the items, by the substitutions of cheaper commodities, or even their complete omission.

In order to illustrate the point referred to, the results of a budget inquiry, in Australia, made in November, 1913, may be taken. This inquiry shewed that on the whole the average weekly expenditure on various items was as shewn in the table hereunder in the second columns, and these, expressed as percentages, are given also in the third columns.

Budget Inquiry, November, 1913. Australia.

Item.	Amount £ s. d.	% on total.	Item.	Am't s. d.	% on total.	Item.	Amn't s. d.	% on total.
Housing ..	0 9 1	12.36	Tobacco ..	0 11	1.25	Medical ..	1 6	2.04
Food ..	1 10 3	41.16	Alcoholic beverages ..	0 10	1.13	Rates & taxes ..	0 10	1.13
Clothing ..	0 10 0	13.61	Fares ..	1 10	2.49	Sports & amusements ..	1 1	1.47
Fuel and light ..	0 3 4	4.53	Insurance ..	2 0	2.72	Charity ..	0 11	1.25
Soap, starch, etc. ..	0 0 10	1.13	Contributions to benefit societies ..	1 3	1.70	Wages ..	0 8	0.91
Other household requisites ..	0 1 2	1.59	Education fees ..	0 8	0.91	Miscellaneous ..	6 4	8.62

Aggregate amount £3 13s. 6d. = 100.00%

This represents the average usage of 392 families scattered over Australia. In the capital cities and larger towns the expenditure on housing is greater than 12 per cent., and may easily reach 20 per cent., consequently the percentage presented by the items has to be readjusted. This example is instructive because it shews that the *essential* feature is really the quantitative one, in this case at any rate. Moreover, when one is dealing with such a question as the minimum "living-wage," it is clear that—in so far as it is an economic possibility—certain of the items are essential to normal healthy life, for example, food, housing and clothing, while other items, however agreeable or even desirable, are unessential—e.g., alcoholic beverages, sports and amusements, etc.

7. **Purpose of price-indexes must be considered.**—These considerations disclose the fact that the object or purpose of a price-index may govern the principles which should guide us in the technique of its determination. We shall shew that definite composite-units of an appropriate character are the only proper bases for exact determinations, though price-ratios and price-indexes, based upon group results, may also be used for more indefinite and less accurate determinations.

It is no longer sufficient to regard a price-index as representing always some general relation between the unit of money and commodities. We shall shew, for example, that although there may be a general price-index representing the relationship between money and all other commodities, this is significant only when their quantitative relations *inter se* are quite definite. Such a price-index, however, may possibly differ sensibly from one appropriate for ascertaining the varying value of the unit of money in regard, say, to the ordinary necessities of life (e.g., a price-index suitable for analysing questions relating to the cost of living). Similarly it will differ from a price-index, the purpose of which is to obtain an idea of the relative quantities of imports or of exports from a record of their values. This must be appropriate to its purpose, and is not always quite satisfactory if it be merely the *general price-index*. Coming down to small communities and individuals, or to various classes within a community, it may be said that there is a price-index appropriate to each, in regard, for example, to their expenditures upon living. Statistical results, however, in order to have any generality of application, must deal with hypothetical "average individuals" or "average communities" as the case may be.

It will be shewn that for any specific purpose whatever a price-index can be quite accurately determined, and that the prolix discussion on methods of ascertaining price-indexes and upon questions of weighting price-ratios in order to obtain them, owe their existence to an inappropriate envisaging of the question.

8. **Exposition of the nature of the technique.**—In Parts II. and III. hereinafter the nature of price-indexes is exhibited, and the appropriate methods of computing them are shewn. In order to exhibit clearly the essential nature of these, the technique of what may be called "extreme cases" is sometimes taken by way of illustration. This has been done because the real significance of particular methods is thus more clearly exhibited. In this way, carefully-selected arithmetical tests become more satisfactory to many than general demonstrations, the nature of which can be followed only by algebraists. Thus—to a certain extent—the latter are rendered unnecessary. Those interested in a general demonstration are referred to Labour Report No. 1 on "Prices, Price-indexes, Cost of Living in Australia," December, 1912. It may here be mentioned that much depends on securing a proper view-point of the whole question (as the well-known controversy between Jevons and Laspeyres shewed: it will suffice here to state that had the question been set out clearly, all differences between these two authorities would have disappeared, as was shewn in the Appendix to the Report mentioned, pp. xxxv. and xxxvi.).

Certain methods, particularly the method of determining price-indexes from price-ratios, appear, on a superficial view, to have much to commend them, because of their apparent generality. This, however, is only in appearance, and when really analysed the apparent merit turns out to be illusory. I shall endeavour to make clear in Parts II. and III. that it is possible to accurately ascertain price-indexes, if the data are available, that they will then be perfectly definite in their meaning; the degree of their applicability can be made manifest; and they can be made quite exact, if only true prices can be ascertained. On the other hand, the method of computing from price-ratios is tedious, and as ordinarily carried out is inexact, while the exact significance of the result so obtained is by no means self-evident.

In Part IV. will be set out such conclusions as have been established in Parts II., and III. In dealing with these questions, the arithmetical examples and method of treatment generally have, as far as possible, been made independent of algebraic exposition.

9. **Differences between household-budgets and composite-units.**—These preliminary observations may be closed by adverting to a wide-spread misconception of the essential character of the method of ascertaining a price-index, in which connection it may be mentioned that the part played by household-budgets is misunderstood.

Public comments from time to time have shewn this. The results of budget inquiries as to the actual cost of living, and the application of the results of investigations to ascertain the fluctuations in the purchasing-power of money are not interdependent. The results of household-budgets may, of course, be used for the purpose of deciding upon the commodities and mass-units of a composite-unit to be employed for the measurement of variations in the purchasing-power of money. It is one of the two possible methods of doing this. They are, however, not essential. The composite-unit may also be determined from general usage; that is, from statistics of consumption. This question is discussed in Part III., Section 6.

The percentage of expenditure upon the different items in a household-budget has often been given as an aid to grasping its significance, and this has unquestionably given rise to an impression—by no means a correct one—that, inasmuch as change in prices disturbs the relative percentages, it necessarily vitiates the deduced price-indexes. This view loses sight of the fact, first, that composite-units may or may not be independent of budget-returns, and that, whatever the basis used may be, the results are sensibly the same, provided the basis is well determined. We have shewn in Labour Report No. 1, and shall later repeat the demonstration, that the basis for the measurement of the fluctuations in the purchasing-power of money is not dependent upon meticulous accuracy as regards a budget inquiry or other research for ascertaining appropriate mass-units for the items of the composite-unit. Though minor deviations of actual usage do not sensibly affect the result, this unit must remain constant. The notion that variations of the relative proportions of expenditure invalidate price-indexes arises only from misconception in regard to the whole matter. The applicability of price-indexes to questions of cost of living is independent of minor deviations therein; in any case the purchasing-power of money cannot be estimated on any other than a constant standard. When the same basis is applied on two occasions the results are sensibly identical, even if the differences of the regime are considerable. But if we use one composite-unit on one occasion and another on the next, we introduce another element, viz., *change of standard-of-living*. There can be no middle course; either we may base the estimate of the cost of living at a particular time upon the actual budgets *at the time*, or we may apply a correction, based upon the fluctuating purchasing-power of money, to a budget ascertained at a particular time, finding in this way the equivalent of the original. That is, we may deduce the cost of living from a previous budget inquiry, or from some other mode of ascertaining what is required in normal living, or we may—on the other hand—ascertain directly what people are actually spending upon living. The two questions are distinct, and have no general relation, one with the other.

The ascertaining of the purchasing-power of money is of wide significance, and virtually presupposes that every person is free to modify his regimen as he pleases, but it is not based upon the ratio of expenditure—an ever-changing quantity—among the particular items in the household budget to the total expenditure thereon. It purports to shew what the general change in the purchasing-power of money is, not by a vain attempt to include all commodities in proportion to their usage, but by restricting the investigation to *identifiable commodities*, so that the result will not be vitiated by uncertain elements that are liable to introduce variations consequent, not upon the change of purchasing-power, but upon change of regimen; that is, change in the *standard* of living.

All attempts to deal with variations in usage, item by item, are open to the criticism that there are actually as many price-indexes as there are individuals, since the usage of one individual is not identical with that of another. The matter must be considered in its generality; thus it is not to the point to shew that any minor item, especially one not definitely identifiable, has changed its price in some other ratio than that indicated by the price-index of the composite unit.

Any practical method of changing wages so as to make the purchasing-power equivalent should of course meet the general case. Instead of stressing an apparent change in any particular item of expenditure as a reason for departing from a well-determined general price-index, it is better to redetermine the actual cost of living from time to time by, say, the household-budget method, and to maintain, for general purposes of comparison, price-indexes based upon the composite-unit method. The whole matter may be set forth in the following way.

When the question is the determination of the actual cost of living, it is essential that an inquiry be made as to the aggregate expenditure upon all items. This, however, having been ascertained for any particular date, price-indexes based upon an appropriate composite-unit may be used for finding its varying money-equivalent, until such a time as the necessity for a further similar budget-inquiry is indicated.

PART II.

THE PURCHASING-POWER OF MONEY AND THE NATURE OF PRICES-INDEXES.

SYNOPSIS.

1. General.
2. The composite-unit as a basis for measuring changes in the purchasing-power of money.
3. Reciprocal comparability of price-indexes at different dates.
4. Accuracy to be expected.
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17. Secular changes in the purchasing-power of money.
18. Mode of ensuring pseudo-continuity.
19. The system of relations between money and commodities.
20. The effect of abnormal times upon price-indexes.
21. Effect of seasonal fluctuations in consumption of commodities.
22. For many questions expenditure is not the measure of importance of commodities.
23. The deduction of purchasing-power from the weighted ratios of rises in price.
24. Recapitulation of the nature of the problem of ascertaining the purchasing-power of money.

1. **General.**—As Prof. Edgeworth has observed (*Econ. Journ.*, June 18, 1918, p. 176), careful measurements of change in the purchasing-power of money will be (and are) required for the adjustment of wages and other payments. It is proposed here to indicate the principles underlying such measurements, and to show that the adoption of a very simple method is both desirable and eminently satisfactory. The essential features of the method are such as to admit of its being readily understood by "the man in the street." Notwithstanding its simplicity it has more to commend it than other methods which—under superficial examination—may apparently be of a more satisfactory character. This simple method of comparing the purchasing-power of money is by ascertaining the cost of a suitably chosen *composite-unit*, the constitution of which we shall later describe.

Before discussing this question it may be said that there is a valid foundation for the instinctive repugnance of mankind to over-subtle methods. The satisfactory solution of a difficulty is often reached, as it were, intuitively, though the complexity of a complete and fully outlined solution would be unintelligible to most, and difficult for any. Were it not so, practical action would often be paralysed or be too long postponed. It is an advantage, therefore, if the method adopted is readily apprehended.

There are three principal ways in which the economical significance of commodities to a community may be measured—

- (i.) By the *quantities it uses* of them ;
- (ii.) By the *amount it spends* upon them ;
- (iii.) By their *utility*, from some particular point of view.

For the essentials of living, the first (or sometimes the third) is of the greatest importance ; for luxuries, the second. Thus the *quantities* of bread, meat, sugar, butter, or fat, etc., are of fundamental importance for healthy life ; that is, they are essentials of existence.

On the other hand, the quantities of gems, jewellery, exquisitely worked fabrics, etc., are relatively of no moment as regards mere existence ; the desire to possess them, and the amount they cost, are the bases of their economic significance, although they are really non-essential to existence.

We have stated that the importance of commodities may also be estimated on other bases, among which one might mention their *food-value*, for example.* This is done by classifying them according to their content in proteins (nitrogenous flesh-forming constituents), fats, and carbohydrates (or sugars and similar substances), and the energy (number of calories) represented by these, compared with the normal requirements of the human body. It is necessary, also, to take account of the suitability (digestibility, etc.) of the food, and of the fact that it contains other constituents (vitamines, etc.), which though apparently negligible in quantity, appear to be essential to proper nutrition.

It may at any time become necessary, through famine or other disaster, to use substitutes for usual foods, in which case the basis of estimation may include other than the ordinary element of price.

Postponing for the present any consideration of this last kind, we note that between commodities of the first two types referred to there are large numbers of commodities that possess intermediate characters, so that in the most general consideration of the nature of commodities we must attribute to them at least two important though opposed characters, viz., necessity (*s*) and non-necessity or unessentiality (*u*). For example, bread, etc., is a necessity ; diamonds are not. If we express these two characters relatively (as ratios to unity) their sum is unity ; that is, we must have—

$$(1) \dots s + u = 1.$$

It is well to remember that even in the same class, individual commodities may possess these attributes or characters in different degrees. For example, in those grades of clothing which are a necessity even to the humblest or most thrifty, *s* is necessarily nearly unity. On the other hand *u* is nearly unity in the case of expensive silks, furs, etc., for they are mainly luxuries. To define more clearly what is meant, let us assume that an overcoat is a necessity, and that there are three grades, the lowest one possible at £3, one at £12, and one at £100. Let us assume also that the one at £12 will be serviceable twice as long as that at £3, and that the one at £100 will be serviceable four times as long. We shall then have, in the first case, $s + u \equiv 1 + 0$. In the second case (disregarding interest questions) the necessity-value, taking account of the duration of its serviceableness, is $2 \times £3 = £6$, and consequently its luxury or unessential element is also £6 ; that is, we shall have $s' + u' \equiv 0.5 + 0.5$. Similarly in the third case we shall have for the necessity-value $4 \times £3 = £12$, and consequently the unessential element £88 ; hence $s'' + u'' \equiv 0.12 + 0.88$. Hence if we are considering the variation in the purchasing-power (*a*) for essentials (*b*) for unessentials, or (*c*) for both combined, we have three different systems of values to take into account. Assuming that the overcoat at £3 lasts two years, we see, for example, that, for the *essentials* of civilised existence, the value per unit of time (1 year, say) is £1 10s. in each case. We observe, in passing, that variations in price ordinarily affect these elements differently, so that the ratio of the two elements (*s/u*) is not at any rate quite constant when prices change. It is not proposed to discuss the measurement of the purchasing-power of money in regard to mere esteem-values or unessential values.

* This might be regarded as a case of utility, (iii.) above.

We notice, also, in regard to (i.) that we may make the basis of comparison (i.a) what people *must use* to maintain healthy and comfortable existence, *in so far as that is possible*, or (i.b) what they *do use*; and similarly the basis may be (ii.a) what they *must spend* and (ii.b) what they *do spend*.

It is also to be observed that we may make the basis of comparison, the usage either as regards quantity or as regards expenditure, that of (1) an individual, (2) a group or class of individuals, or (3) an entire community, a people, or an empire, etc. Which we do will depend upon the *purpose* we have in view. We shall consider later what would be obtained in the several cases, remarking, however, that—speaking generally—usage according to quantity is satisfactory, and according to expenditure unsatisfactory.

2. The composite-unit as a basis for measuring changes in the purchasing-power of money.—Suppose, to take a homely illustration, that a thrifty housewife made out a list of her regular marketing requirements, say a list of the things she must purchase each week. Against this list she jots down what she spends on each item from week to week, and totalling these, sees what her requirements cost in the aggregate. The *aggregate-expenditures* then are the cost, not of any one thing, but of the whole series of things, not of any one unit (1 loaf, 1 lb. of meat, or what not), but of a week's total requirements. These aggregates of expenditure would reveal to her exactly how far £1 would go. For example, if they cost at one time 60 shillings and at a later date 80 shillings, it is clear that £4 at the later date goes only as far in purchasing the series of things constantly required as £3 went formerly. The *index of this* is that for every £1 formerly required, £1 $\frac{3}{4}$ are required at the later date. We could, of course, put it in another way, viz., that, in regard to her requirements, the *purchasing-power* of £1 of money has fallen to 15s., that is, in the ratio of $\frac{3}{4}$ to 1, or of 1 to $\frac{4}{3}$. This system of estimating is the only one which is quite flawless in principle. It is based upon the *aggregates of expenditure for a fixed series of commodities*. Instead of expressing this index by the number 1 $\frac{3}{4}$, we could multiply it by 100 or 1000, etc., when we should have 133 $\frac{1}{3}$, 1333 $\frac{1}{3}$, etc.

Let us restate this: we consider first the case of an *individual* whose *usage* is *constant*, whose *wants* are of the same nature, and who has decided that he will not—under any circumstances—vary the quantity of the commodities which he requires. In such a case he could proceed as follows:—He could write out a list of commodities, the amount of each he used, and the price he had to pay. Then, multiplying the quantities by the prices paid and adding the various sums, the total amount would be the *aggregate expenditure* for his list. This list of commodities, with the quantities for each item, we can call the *constant composite unit*, and the amount paid for it the *cost of the composite unit*. Now for such a person the *purchasing-power of money* would vary reciprocally as the cost of the composite-unit. Symbolically this may be set out as follows:—

List of commodities in unit	..	A	,	B	,	C	,	D	,	etc.
Quantity of commodities in unit..		q_a	,	q_b	,	q_c	,	q_d	,	etc.
The composite-unit itself is:—U	=	q_a of A	+	q_b of B	+	q_c of C	+	q_d of D	+	etc.
Prices of items in unit (per unit-quantity)..	..	P_a	,	P_b	,	P_c	,	P_d	,	etc.
Cost of items in unit	..	$q_a P_a$	,	$q_b P_b$	,	$q_c P_c$	,	$q_d P_d$	,	etc.
Cost of composite-unit, P (say)	=	$q_a P_a$	+	$q_b P_b$	+	$q_c P_c$	+	$q_d P_d$	+	etc.

Suppose, then, that at some particular time, adopted as a date of reference, the cost of U is ascertained to be P_0 , and at other dates was found to be P_1 , P_2 , etc. (it is immaterial, of course, whether in point of time these be earlier or later): then the *purchasing-power of money* will have fallen if P_1 , P_2 , etc., are greater than P_0 : it takes a greater sum of money to purchase the composite-unit at dates 1 and 2 than it did at date 0. Suppose, for example, that at date 0 it cost £8 to purchase the composite-unit, and at dates 1 and 2 it cost £10 and £12 respectively; then clearly the purchasing power has fallen from—

$\frac{8}{10}$ to $\frac{8}{12}$ (at date 1) and to $\frac{8}{12}$ (at date 2), or from 1 to $\frac{4}{5}$, and then to $\frac{2}{3}$.

Or if, for convenience, we make the first 1000, we shall have the purchasing-power represented by the numbers—

At date 0, 1000; at date 1, 800; at date 2, 666 $\frac{2}{3}$.

the last result being 667 if expressed to the nearest integer. These numbers 1000, 800, and 667, may be called *indexes of the purchasing-power of money*.

Suppose, however, that instead of so expressing these results, the comparison is made in the form which shews how much is necessary to purchase a definite quantity of the composite-unit, making the *price at date 0, 1000* (i.e., 1000 pence, shillings, pounds, or any other unit): the numbers would then be *price-indexes*. Thus we should have for the three cases above—

$\frac{8}{8} \times 1000$ (at date 0) = 1000; $\frac{12}{8} \times 1000$ (at date 1) = 1250; $\frac{15}{8} \times 1000$ (at date 2) = 1500.

These three numbers, 1000, 1250, and 1500, are the *price-indexes* for the dates in question. They shew how much money is needed to purchase a certain commodity-unit, and if this unit be well selected, they shew how the purchasing-power of money generally is rising or falling, viz., by the falling or rising respectively of the price-index. If the *cost* is rising the *purchasing-power* is of course falling.

3. **Reciprocal comparability of price-indexes for different dates.**—It is obviously desirable that we should be able readily to change our basic date. For example, if in the preceding instance we wished to make date 1 or date 2 the basic date, instead of date 0, our indexes should give us the same relations as before. We shall call the dates 0, 1 and 2, 1916, 1917, 1918 (for convenience). Thus we must have:—

Basis.	Data.	Price-indexes.
With 1916 as basic year ..	£8 : £10 : £12 =	1000 : 1250 : 1500.
With 1917 as basic year ..	£8 : £10 : £12 =	800 : 1000 : 1200.
With 1918 as basic year ..	£8 : £10 : £12 =	666 $\frac{2}{3}$: 833 $\frac{1}{3}$: 1000.

Let us suppose that we were given the price-indexes on the 1916 basis, and wished to change them so as to make 1918 the basic year: knowing nothing of the actual cost of the composite-unit from which the indexes were found, we have to find the values of:—

For 1916, $\frac{1000}{1500} \times 1000 = 666\frac{2}{3}$; for 1917, $\frac{1250}{1500} \times 1000 = 833\frac{1}{3}$;
for 1918, $\frac{1500}{1500} \times 1000 = 1000$.

that is, we get exactly the same results as if we have worked with the original figures $8 \div 12 \times 1000$; $10 \div 12 \times 1000$; $12 \div 12 \times 1000$. Simple and obvious as this may appear, it should be noted that price-indexes have not always been found in such a manner that they possess this property of being independent of the year selected as basis.

Not to burden the illustration unnecessarily, let us suppose, for example, that the composite-unit consisted only of three items, A, B and C, costing in 1916 respectively £1, £2 and £5, i.e., £8 in all. One method of attaching importance to these is to weight them in the ratio which the expenditure on each bears to the total expenditure. Thus in 1916 the weights were the ratios $\frac{1}{8}$, $\frac{2}{8}$ and $\frac{5}{8}$. Having found their weights, it is usual to *weight* all future results accordingly. Thus if later (in 1917) they cost respectively £2, £2, and £6 (£10 in all) and still later (in 1918) they cost respectively £2 10s., £2 10s. and £7, the procedure in calculating the price-indexes is as follows:—

Commodity.	Price. 1916.	Price. 1917.	Price. 1918.	Price-Indexes of each Commodity (1916 as basic year).		
				A	B	C
A	1	2	2 $\frac{1}{2}$	1000	2000	2500
B	2	2	2 $\frac{1}{2}$	1000	1000	1250
C	5	6	7	1000	1200	1400
Total A+B+C	8	10	12	Simple mean 1000	1400	1716 $\frac{2}{3}$

The simple means (for equal weights), viz., 1000, 1400, 1716 $\frac{2}{3}$ (instead of 1000, 1250 and 1500) are evidently enormously in error. When, however, instead of three, a large number of commodities is taken, this error relatively diminishes but does not disappear; it is rarely insensible, and then only fortuitously so. Suppose, then, that for any date subsequent to 1916, we weight the price-indexes for the individual items by multiplying them by the factors $\frac{1}{8}$, $\frac{2}{8}$ and $\frac{5}{8}$, the sum of which is unity. Then we have :—

$$\begin{array}{rcl} \text{For 1917—} 2000 \times \frac{1}{8} & = & 250 \\ 1000 \times \frac{2}{8} & = & 250 \\ 1200 \times \frac{5}{8} & = & 750 \\ \hline \text{Total (1)} & = & 1250 \end{array} \quad \begin{array}{rcl} \text{For 1918—} 2500 \times \frac{1}{8} & = & 312\frac{1}{2} \\ 1250 \times \frac{2}{8} & = & 312\frac{1}{2} \\ 1400 \times \frac{5}{8} & = & 875 \\ \hline \text{Total (1)} & = & 1500 \end{array}$$

Similarly, if we adopt any other as the basic year, and base the weights upon the *relative expenditures of that year* (e.g., 1917 = $\frac{1}{10}$, $\frac{2}{10}$ and $\frac{7}{10}$ or $\frac{1}{5}$, $\frac{2}{5}$ and $\frac{3}{5}$; or 1918, $\frac{1}{10}$, $\frac{2}{10}$ and $\frac{7}{10}$), we shall get the correct results *because* the process merely reproduces—in a very roundabout way—what we should have obtained by adding the cost of the items,* finding the ratios to the first, and multiplying by 1000.

It has been—strangely enough—imagined that the difficulty about relative quantities was disposed of by taking the mean of the price-indexes of a series of commodities, whereas in reality it was merely hidden. Thus the logic of the method was that by merely observing how prices changed (say from 1000 to 2000, to 2500, etc.) we might escape any detailed examination of the extent to which each index should be allowed to influence the general result. Thus no account was taken of the fact that the relative expenditure changes with all non-uniform variations of price as among the several items in the composite unit. Thus let us suppose we have decided to use weights $w_a = \frac{1}{8}$, $w_b = \frac{2}{8}$, and $w_c = \frac{5}{8}$, and we work back from the 1918 results (which are to be taken as 1000) to find the 1916 and 1917 price-indexes. We have then :—

Commodity.	1916 Indexes.	Index ;	Weight.	1917 Indexes.	Index ;	Weight.
A	$\frac{1}{2\frac{1}{2}} \times 1000 = 400$;	$\times$	$\frac{1}{8} = 50$	$\frac{2}{2\frac{1}{2}} \times 1000 = 800$;	$\times$	$\frac{1}{8} = 100$
B	$\frac{2}{2\frac{1}{2}} \times 1000 = 800$;	$\times$	$\frac{2}{8} = 200$	$\frac{2}{2\frac{1}{2}} \times 1000 = 800$;	$\times$	$\frac{2}{8} = 200$
C	$\frac{5}{7} \times 1000 = 714\frac{2}{7}$;	$\times$	$\frac{5}{8} = 446\frac{17}{56}$	$\frac{6}{7} \times 1000 = 857\frac{1}{7}$;	$\times$	$\frac{5}{8} = 535\frac{5}{7}$
	$\left(\text{Mean} = 638\frac{2}{21} \right) ; I = 696\frac{17}{56} \left(\text{Mean } 819\frac{1}{21} \right) ; I = 835\frac{5}{7}.$					

The correct indexes are, for 1916:— $8 \div 12 \times 1000 = 666\frac{2}{3}$; and for 1917:— $10 \div 12 \times 1000 = 833\frac{1}{3}$; instead of which we get the erroneous results $696\frac{17}{56}$ and $835\frac{5}{7}$. The reason of this is obvious; we have not used the weights which the commodities possessed in the basic year, viz., $\frac{1}{2\frac{1}{2}}$, $\frac{2}{2\frac{1}{2}}$ and $\frac{5}{7}$; it is easily verified that if we had we should have obtained the true results. Thus if we substitute the proper weights for those previously used, we get the correct results, thus :—

$$83\frac{1}{3} + 166\frac{2}{3} + 416\frac{2}{3} = 666\frac{2}{3}; \text{ and } 166\frac{2}{3} + 166\frac{2}{3} + 500 = 833\frac{1}{3}.$$

But, as previously stated, the process is indirect and involved, and we do not see what we are doing.

* This is readily seen if we shew in detail the elements of the calculation. Thus, I denoting the price-index, the process for finding it for 1918 is :—

$$I = \frac{1}{3} \left(\frac{2\frac{1}{2}}{1} \times 1000 \right) + \frac{2}{3} \left(\frac{2\frac{1}{2}}{2} \times 1000 \right) + \frac{5}{3} \left(\frac{7}{5} \times 1000 \right) \div \left(\frac{1}{8} + \frac{2}{8} + \frac{5}{8} \right).$$

Multiplying both numerator and denominator by 8, we have—

$$\begin{aligned} I &= 1 \left(\frac{2\frac{1}{2}}{1} \times 1000 \right) + 2 \left(\frac{2\frac{1}{2}}{2} \times 1000 \right) + 5 \left(\frac{7}{5} \times 1000 \right) \div (1 + 2 + 5); \\ &= \left\{ \left(2\frac{1}{2} + 2\frac{1}{2} + 7 \right) \div (1 + 2 + 5) \right\} \times 1000 = 1500. \end{aligned}$$

4. **Accuracy to be expected.**—If we express results to the basic index of 1000, it is implied that the error is not as much as a half-unit either way; that is to say, the index is greater than 999.5 and less than 1000.5. To get such a degree of precision the aggregate cost of the composite-unit adopted must be of a still higher order of accuracy. Suppose, for example, its value at one date is about £8 = 1920 pence, and on another about £8 17s. 0½d., or 2124½ pence. If the first is the basic date this would give an index of 1106.51, which would be written 1107. Suppose the true amounts were £8 0s. 1½d. and £8 16s. 10½d., the index would then be 1104.49, which would be written 1104. The difference is thus 3 units. The total errors of price causing this are 3½ pence. Roughly we may say that the order of accuracy in ascertaining price must be about 1 in 2000 to ensure a precision of 1 in 1000. It is obvious that prices must be well ascertained in order to reach this order of accuracy.

5. **Price-indexes for an individual.**—If an individual, whose requirements are (or may be deemed to be) *constant*, were to keep records of his expenditure, his list of items would include expenditures for food and groceries, for rent or its equivalent, for boots and clothing, for travelling expenses, books, and other educative expenditure, for luxuries and amusements, for contributions to insurance of various kinds, for medical and similar attendances, and so on. For short periods, say of the order of a quinquennium, or even a decade or two, the general trend of human affairs may be regarded as fairly constant, but the purchasing-power of money is ever fluctuating. In view of the fact that a considerable number of commodities are continuously available to serve as the basis for ascertaining this fluctuating purchasing-power of money, it is evidently only a matter of keeping proper record to obtain an *unequivocal measure* thereof. Hence the individual would have to select a suitable group of commodities to serve as a basis for estimation. It is self-evident that the best would be that which represents his average needs. To do this with accuracy, the observations of his requirements must embrace a sufficiently long period to obtain a tolerably accurate average; but we shall shew later that such an average need not be as accurate as the record of the varying price of the commodities. So long as this list of items constituting the composite unit represents substantially the usage of the commodities, and so long as the prices paid for such commodities are accurately recorded, so long will the determination of the price-index be satisfactory. The items and the quantities adopted must, however, be identical for any dates that are to be compared. If they are not identical, we do not get an unequivocal measurement of the purchasing-power of money, but the joint effect of a variation of the items (either as to quantity, or as to the actual commodity, or to both combined). We may call this variation a *change of regimen*, i.e., a change in the composite-unit. For example, if the individual changes the grade of the things he uses, or the items themselves, he can no longer make a comparison as to the purchasing-power of money, although, of course, it in no way hampers him as regards ascertaining what he is spending upon his living.

6. **Effect of changes in the composite-unit.**—There are two ways in which the composite-unit may be changed, viz., (i.) by changing the relative proportions of the commodities used; (ii.) by changing the commodities themselves. The second might appear to be a more radical change than the first, but from the standpoint of the determination of price-indexes it is hardly less so. If, however, we have the prices for a series of commodities, it is easy to ascertain the consequence of any variation of the mere relative quantities which go to compose the composite-unit. On the other hand, if the past record of the new items is not available, the ascertaining of the price-index for the earlier dates ceases to be a possibility. In order to bring into clear relief the equivocal effect of any radical change in the composite unit, let us suppose that at two different dates an individual's actual usage is different, being first U_1 and then U_2 , composed, say, as follows:—

$$(2) \dots U_1 = a \text{ of } A + b \text{ of } B + c \text{ of } C + d \text{ of } D + \text{etc.}; \text{ and}$$

$$(2a) \dots U_2 = q \text{ of } Q + r \text{ of } R + s \text{ of } S + t \text{ of } T + \text{etc.}$$

Obviously no direct comparison is now possible. That the individual might, for example, appropriate his entire income to the purchase of U_1 , and then afterwards to the purchase of U_2 , shews that we cannot discover the relative purchasing-power. If, for the price-indexes, we used composite-unit U_1 on both occasions, or composite-unit U_2 on both occasions, we should obtain two different results for the purchasing-power of money.

Thus it might be said, "If one had continued to use U_1 , the purchasing-power of money would have changed in such-and-such a way," or, if one had originally used U_2 , then the change in the purchasing-power would have been in such another ratio.

But neither estimation would really be applicable: both would be purely hypothetical cases. There is no escape from a supposititious case if we are to obtain any indication at all. The only practical solution of real value is obtained by *adopting a hypothetical regimen* or composite-unit, which would occupy, as near as could be judged, a sort of middle position between the two, say—

$$(2b) \dots U_m = g \text{ of } G + h \text{ of } H + i \text{ of } I + j \text{ of } J, + \text{ etc.}$$

This hypothetical unit is then used to measure the change; or one could (in a rougher way) *arbitrarily* take the mean of the two determinations, the one being based upon U_1 , and the other based upon U_2 .

The most general and satisfactory solution, however, is to include in U_m all the items in U_1 , and U_2 ; that is, we should make it consist of a of A , plus q of Q , plus b of B , plus r of R , and so on. (We need not divide these by 2, because the magnitude of the unit does not affect the case, as already shewn). That is, we constitute a fictitious composite-unit, one half of which is true for one period, and the other half for the other, in order to get a comparison between the two. This fictitious unit thus furnishes some basis for a comparison, but strictly is inapplicable. We shall return to this question later and give it more extended consideration.

7. **Meaning of price-indexes for groups of individuals.**—In order to fix our ideas, let us picture a relatively small group, say of 1000 persons, whose general usage of commodities was much the same. If now, *in order to embrace all seasonal variations*, we ascertain their total consumption of commodities during a period of 12 months, this will constitute a composite-unit for the group, and one-thousandth part of this will be the *average consumption* per person per annum for the entire group, although possibly not one of the 1000 persons would consume exactly that amount. We may then say that, *for practical purposes*, we are entitled to assume that this average consumption applies not only to the group (which of course it does, quite strictly), but *also to the individual members, for all general purposes*. It is only by means of some such hypothesis that *any price-index has validity* for the individuals of the group. The composite-unit for a group, then, is that representing the group usage. The group may be regarded either as an individual, or as a number of individuals having a common usage. For most purposes it is a matter of indifference which we suppose: it is only when we have occasion to distinguish between individual and average usage that the matter becomes of moment.

8. **Relative, not absolute, amounts necessary to constitute the composite-unit.**—It has already been mentioned that when we ascertain separately the quantities used by two individuals and combine the two results we need not divide by 2. Either the unit of time for which we ascertain average consumption, or the exact number of persons for whom it is ascertained, is of no moment excepting to ensure that the relations between the average quantities used are correct. Hence it is clear that we need know only their *relative* amounts to properly constitute the composite-unit. We are not concerned whether these amounts are per day, week or year, nor whether one individual consumes two average composite-units and another only half of one. What we *do* require to know is how much of each commodity is used *on the average* in any unit of time whatsoever, or for any number of persons whatsoever (the same throughout, of course). The ratio of the cost of this unit at any two dates is obviously the same whatever its size, and depends solely upon the relations of the individual items among themselves, and their prices. If these ratios are the *ratios of actual usage* and the unit is complete, the method is flawless: the change of purchasing-power is fully and exactly ascertained by attributing the actual total cost on the dates to be compared. It is also self-evident that there is no essential difference between the comparisons of price-indexes for two individuals at different places at the one date, or for one individual at two different dates at the one place.

9. **Comparability and non-comparability of individuals and communities.**—If the commodity-usage of two individuals or two communities is identical, then not only are their price-indexes comparable, but also their actual expenditures for equivalent amounts of the unit. With communities whose commodity-usage is entirely

different, a series of price-indexes are strictly non-comparable; the actual expenditure cannot be compared except on the basis of its absolute amount. For example, let us suppose two communities, in one of which the staple food was rice and fish, and in the other wheaten bread and beef, to exist; and to simplify the illustration of the principle governing comparability, let us restrict ourselves to these two items. The variations of purchasing-power are then measured in the one case by the prices of rice and fish, and the relative amounts used of these; in the other by the price of bread and beef. These prices have no necessary connection, and hence the variations in the price-indexes of one community *have no application in the other*. Similarly, among individuals, if the diet and mode of life of one be simple and severe, including but few classes of food and those the cheapest, and that of another be elaborate and luxurious, including great variety and expensive foods, the price-indexes appropriate for the former have no validity for the latter, or *vice versa*. Each is concerned only with the variations in the commodities he uses, and the variation of the purchasing-power of money is wholly dependent upon the applicability or otherwise of the regimen. *The composite unit must be that of actual usage in order to have intelligible meaning and to be applicable in the world of fact.* In so far as it does not represent actual usage it is meaningless.

As before, let U_1 and U_2 denote two different composite-units. It will sufficiently illustrate the point if we take, say, a group of 4 commodities, with their corresponding quantities and prices.

Composite Unit U ₁						Composite Unit U ₂					
Commodity. (1)	Units and Weig ^t used. (2)	Price and Aggregate Expenditure.				Commodity. (1)	Units and Weig ^t used. (2)	Price and Aggregate Expenditure.			
		(3) 1912.	(4)	(5) 1917.	(6)			(3) 1912	(4)	(5) 1917	(6)
Bread ..	lb. 80	*1.5	†120.0	*1.75	†140	Rice ..	lb. 35	*2.80	†98.0	*3.1	†108.5
Beef ..	lb. 75	5.1	382.5	8.7	652.5	Fish ..	lb. 300	1.50	450.0	2.0	600.0
Butter ..	lb. 10	15.7	157.0	18.2	182.0	Sugar ..	lb. 46	2.90	133.4	3.5	161.0
Coffee ..	lb. 3	18.6	55.8	19.0	57.0	Tea ..	lb. 3	14.70	44.1	17.5	52.5
Totals ..			715.3	1031.5		Totals ..			725.5	922.0	
In Ratios		(Price-indexes)	1000	1442.0		In Ratios ..		(Price-indexes)	1000	1270.8	

* Price per unit shewn in pence. † Aggregate expenditure shewn in pence.

The results shew that the aggregate for 1912 was 715.3 for composite-unit U_1 , and was 725.5 for composite-unit U_2 , but established nothing as to the relative purchasing-power of money in the two places, *because there is no common basis for estimating this*.

If the units in composite unit U_1 , and those in composite-unit U_2 happen to represent the average consumption for an equal period (which is not at all necessary when they are used for ascertaining price-indexes), then all we know is that 715.3 pence in the former case would correspond to 725.5 in the latter (that is, of course, if this list of items in the regimen were complete, which obviously it is not).

The change in the purchasing-power of money is properly found and reciprocally shewn in the price-indexes for individual—or for community—(1), these being 1000 and 1442.0, and for individual—or community—(2) being 1000 and 1270.8. It may be asked—“On what basis could we compare the two places in this respect?” A decline in the purchasing-power of money is certainly exhibited in both results but it is not identical. “Can there be a general representation shewing the joint result?” may be asked. Applying the principle indicated in section 6, we can obtain a kind of comparison by supposing *that each changed its regimen so as to include that of the other*. It is now essential that the units in U_1 and U_2 should be for identical periods and numbers of persons. Let us suppose that this is the case. Then we can reconstitute the series so as to contain the whole. We should then have for our grand totals 1440.8 for 1912 and 1953.5 for 1917, giving the price-indexes say 1000 for 1912, and thus 1355.8 for 1917. Had we simply taken the mean of the two results 1442.0 and 1270.8, we should have obtained 1356.4. The two results, though fortuitously (and usually in practical cases) very nearly the same, are not of course identical.*

* The prices have been taken as though those in the supposititious community were identical with the prices in the two communities.

The interpretation of these results is important. Either result by itself represents the change in the price-index for that community, on the assumption that all the items used (and sensibly influencing the result) are taken account of in columns (1) and (2), the prices being as in columns (3) and (5), giving the aggregates of expenditure as shewn in columns (4) and (6). The two results have, as already mentioned, no direct connection with each other. They shew, however, that the purchasing-power for community (1) *with its own regimen* has fallen in the ratio 1000/1422.0, that is, from 100 per cent. to 69.348 per cent., and in community (2) also *with its own regimen* in the ratio 1000/1270.8, that is, from 100 per cent. to 78.681 per cent. When we combine them in one total, on the basis of the composite-unit U_m (the sum or mean of both), obtaining the price-indexes 1000 and 1355.8, it implies that for a community whose average usage embraced all the items of the two and *in identical proportions* (not absolute amounts), the fall in the purchasing-power would have been in the ratio 1000/1355.8; that is, from 100 per cent. to 73.757 per cent. Such an hypothetical community could be constituted by combining equal numbers of each, viz., of community (1) and community (2) with prices changing as shewn.

Suppose, however, that community (2) was only one-tenth the size of community (1). The composite-unit for the aggregate of the two communities combined would be the quantities in the left-hand side of the table plus one-tenth of those in the right-hand side. Thus we should have:—

Commodity.	Bread.	Beef.	Butter	Coffee	Rice.	Fish.	Sugar.	Tea.	Total.
Units lbs.	80	75	10	3	3.5	30.0	4.6	0.3	..
Price 1912	1.5	5.1	15.7	18.6	2.8	1.5	2.9	14.7	..
Price, 1917	1.75	8.7	18.2	19.0	3.1	2.0	3.5	17.5	..
Expenditure—1912	120.0	382.5	157.0	55.8	9.8	45.0	13.34	4.41	= 787.85
„ 1917	140.0	652.5	182.0	57.0	10.85	60.0	16.10	5.25	= 1123.70

These aggregates of expenditure for 1912 and 1917 shew that the price-indexes were 1000 and 1426.3: that is, the purchasing-power of money had fallen from 100 per cent. to 70.110 per cent.

We could get *approximately* the same result by merely weighting the results for communities (1) and (2) according to their populations 10 and 1, total 11. The 1000 for 1912 is of course unaffected. For 1917 we have $\{(1442.0 \times 10) + (1270.8 \times 1)\} \div 11 = 1426.4$, instead of 1426.3 the correct amount. We saw earlier that where the populations were assumed to be equal, we also got approximately the same result; that is $\{(1442.0 \times 1) + (1270.8 \times 1)\} \div 2 = 1356.4$, instead of the 1355.8, the correct number. If we were to shew the price-indexes to four places of figures only we should have identical results.

The results, when interpreted properly, are *quite definite* and may throw light upon an important phenomenon; for example, the world-wide falling in the purchasing-power of money. For *that* particular purpose they would be appropriate, notwithstanding their non-comparability for other purposes.

10. Price-indexes for a class, a community, a state, or an empire.—Different classes of a community have characteristic differences in their habits of eating and living which are reflected in the commodities they require, and in the relative proportions subsisting between the commodities which they use in common. Thus the composite-unit appropriate to each is not quite identical, and therefore comparisons between the two are not directly possible. If for any *special* inquiry it is required to distinguish between the purchasing-power of money for different classes, communities, States, etc., the appropriate composite-units must be employed.

Among peoples, where the great mass are occupied in ordinary business avocations, the composite-unit can be based upon the usage of the entire community; the elimination of the effect of particular classes has no sensible effect upon the numbers for the whole. The ascertaining of the average usage of the entire community has the advantage also of generality; it founds the comparison on the

average man of the whole community, whereas the other is the average man only for the class in question. A similar remark applies also to the various States of a Commonwealth or of an Empire. The most general basis is the average usage for the whole.

Domestic questions, however, may render it necessary to have also composite-units for each different State, and even for the capital cities as differentiated from the States to which they belong. The criterion is whether the discrimination introduces any sensible change into the result. If it does not, then the most general composite-unit is the best. In any case if mutual comparability is desired, the basis must be uniform.

11. **Small effect of considerable differences in the composite-unit.**—In order to shew that quite considerable changes in a composite-unit do not enormously affect the price-indexes determined by means of them, we will take actual cases which have been calculated on ruling prices. In these cases (I.) denotes the quantities used in the official publications of the Commonwealth Bureau of Census and Statistics; (II.) denotes those used by the English Board of Trade, 1904; (III.) denotes quantities advocated by certain workers in the Northern Territory of Australia as typical of the weekly consumption of a working class family consisting of a father, mother and two children; (IV.) is the major part of a dietetic scale which has been suggested.*

Various Composite-units or Regimens, I. to IV.

Com- modity.	(I.)	(II.)	(III.)	(IV.)	Com. modity.	(I.)	(II.)	(III.)	(IV.)
	lbs.	lbs.	lbs.	lbs.		lbs.†	lbs.†	lbs.†	lbs.†
Bread ..	21.1	22.0	18.0	20.0	Potatoes ..	20.2	17.0	7.0	14.0
Flour ..	6.7	10.0	1.0	0.5	Onions ..	1.5	—	5.0	—
Tea ..	0.7	0.6	1.0	0.5	Milk ..	6.8 qts.	5 qts.	6 tins*	7 qts.
Coffee ..	0.05	—	—	—	Butter ..	2.1	2.0	1.5	2
Sugar ..	10.4	5.33	6.0	4.0	Cheese ..	0.3	0.75	—	—
Rice ..	1.1	—	—	—	Eggs ..	4.9†	12.0	15.0	—
Sago ..	0.2	—	—	2.0	Bacon ..	0.8	—	—	—
Jam ..	1.6	—	3.0	1.0	Ham ..	0.2	1.5	—	—
Oatmeal ..	0.8	—	—	3.3	Beef ..	8.7	—	—	—
Raisins ..	0.3	—	—	1.0	Mutton ..	7.5	6.5	15.0	16.0
Currants ..	0.3	—	—	—	Pork ..	0.8	—	—	—

— denotes omitted in regimen. * Large tins. † lbs. except where otherwise shewn.
‡ Number of eggs.

When these different composite-units were used and the prices of 1912 and 1915 were applied, the following results were obtained, the 1912 price-index being 1000 :—

(1) 1253; (2) 1255; (3) 1254; (4) 1223; (5) 1228; (6) 1232; (7) 1243;
(8) 1288; (9) 1247.

These were as follows :—§

- (1) Complete regimen as adopted in the Australian Bureau of Census and Statistics;
- (2) Regimen (I.) omitting eggs; (3) Regimen (I.) but omitting butter;
- (4) Regimen (I.) but reducing the consumption of meat to one third;
- (5) Regimen (I.) with reduction as in (4), but also an increase of one-third in bread;
- (6) Dietary scale II.; (7) Dietary scale III.; (8) Dietary scale IV.;
- (9) Average of the preceding eight results.

It is thus seen that (1), (2) and (3) are sensibly identical, the range being less than one half-penny per pound sterling; they are almost in agreement also with the average (9). The marked change implied in (4) and (5) and dietary scales II. and III. (results (6) and (7)) has *not* a marked influence on the result. The reason of this is that the effects of changes are only differential.||

* By Richard Arthur, M.D., M.L.A., New South Wales.

§ The detailed calculations are not here given.

|| Suppose, for example, the original aggregate expenditure is 2076, made up of various items, and the aggregate on the second date is 2589. This gives a ratio of 1000 to 1247.1. And suppose also items are omitted from the regimen, reducing the first aggregate of expenditure by say 200, the cost of these being only 211 at the second date (1000 to 1055). The amended figures would be 1876 and 2378, their ratio being as 1000 to 1267.6 instead of 1247.1; thus the effect is very slight. If the advance had been 200 to say 249, the result would have been sensibly as before, i.e., 1876 to 2340, or 1000 to 1247.3, because the advance was in the same ratio as the balance of the items.

12. **The relation between price and the composite-unit.**—In ascertaining the cost of a composite-unit we are concerned solely with its *exchange-value* under the ordinary conditions of sale and purchase. Hence, in comparing the cost at two different dates, we must see that the standard or quality of the commodity, or anything influencing its exchange-value, is identical on the two occasions. Its intrinsic, esteem, utility, or other value is of no moment. If, for example, we compare a cheap and poor quality of tea at one date with a dear and high quality at another date, the resulting higher price-index for the latter is due (at any rate in due proportion) to this difference of quality being reflected in the price. The essence of the comparison is that the prices of the commodities are solely the variable element. Thus if (as tastes alter, or as circumstances are deemed to warrant a change in the grade of commodities used) a change is actually made, the variation so caused is *not due to a variation in the purchasing-power of money, but to a change in the standard of usage of commodities*. This may be called change in the *standard-of-living*. Variations of this standard may easily enter into and vitiate the estimations of purchasing-power, and is perhaps the most serious practical difficulty in accurately ascertaining variations in the purchasing-power of money. We shall deal at length with this question hereinafter. It must suffice here to set forth the principle which governs accurate determination, which is:—The grade must be constant, and the price the sole variant.

13. **Criteria of constancy in the standards of quality of commodities.**—Assuming that the quantities of a composite-unit have been well ascertained, there still remains the necessity of seeing that their grade or standard is maintained constant if they are to be used to measure accurately the purchasing-power of money. Certain commodities can be specified very readily in this respect; that is, the grade or quality may be regarded as definitely the same in each locality, and for each successive date. For example, among foodstuffs, commodities like bread, flour, cereals of various kinds, sugars, milk, butter, eggs, etc., can be so described as to ensure reasonably accurate identification of quality. On the other hand, textiles, clothing, boots, hats, articles of attire, etc., are by no means easy to define as to quality. Where a commodity is so difficult to identify in respect of grade that the uncertainty introduced by its inclusion among the items of a composite-unit is sensible, then of course it must be excluded from the list of commodities used for the purpose of measuring change in the purchasing-power of money. It is, of course, obvious that two questions converge here. If an available commodity or grade of commodity disappear, and a higher and more expensive grade is alone available, one is compelled to a greater expenditure; this is a question of change in the standard of living rather than a question of change in the purchasing-power of money; it is perhaps a change in the purchasing *opportunity* of money. Questions of this kind are material in estimations of the *cost-of-living*.

14. **Absolute maintenance and quasi-continuity of standards.**—We are here face to face with a real difficulty in regard to commodities. When the question is closely scrutinised it is seen that *the notion that there is or can be any absolute constancy of standard is based upon a fiction*. In foodstuffs, for example, sugar as made to-day is almost chemically pure sucrose: it is a notable instance of the commercial production of a chemically high-grade substance. The last half-century has seen many changes in this respect. Many grades of steel did not exist till recently. Textile fabrics are totally different from what they were a few years back. Since the introduction of bootmaking-machinery certain types of boots have practically disappeared from many communities. Cereals, and products manufactured from them, are prepared in a greater variety of forms. "How then," it may be asked, "is it possible to maintain the standard; and, if in principle that be essential, how can there be any hope of accurately measuring the purchasing-power of money? Must we not also connote the idea of *purchasing-opportunity* in 'purchasing-power,' make the term more elastic, and adapt it to the facts of human life rather than to the expression of an impossible and merely theoretical ideal?"

The answer to these questions must, of course, be that the maintenance, over decades and centuries, of a standard, grade or quality, is impossible; (even commodities themselves change) that such a proposal would be futile; and a solution, depending upon such a hypothetical course, valueless. There must, therefore, be a continual revision of the constitution of the composite-unit so that it may always represent the usage of mankind. Probably this revision should be made every 10 years, so that a sort of quasi-continuity may be thus established. A real continuity

is impossible. There is no basis by means of which we can justly compare the purchasing efficiency of money in the days when "gobbets" of meat were picked out of a dish with the fingers, with to-day with its dining-table furniture; or the days when the clavier, harpsichord or spinet was the instrument in vogue, with to-day with modern instruments of the same type. Obviously we might inquire how much labour was needed to live according to the standard of those earlier days, and how much labour is now required to live according to present day standard, *but that is another question*. The hope of continuity is futile: it must be sufficient to establish what may be called a quasi-continuity, viz., one which, though fairly satisfactory for comparisons covering only a few decades, cannot but be less and less significant as the interval becomes greater. *To expect more is to ask for the impossible*. We shall have to become accustomed to the conception that while we may have results numerically expressed covering long periods, and while these may be very satisfactory for points of time at all near together, yet these apparently precise results *inevitably lose their significance or become non-interpretable as the intervals increase*. This is the fact which it is necessary to keep clearly in mind if we are not to becloud ourselves with illusions about the nature of comparisons of the kind under review.

15. Nominally constant standards.—There is another difficulty in regard to grade or standard of the items of a composite-unit, which has some slight analogy to the one just mentioned, viz., that touching real identification of grade or quality or even the character of the article itself. A commodity like sugar, for example, may well be of the same grade in all places. But commodities like tea and coffee are of many grades, the identification of which, even if they were practically definable, is virtually impossible. Again, fuel may consist of wood, lignite or brown coal, anthracite or black coal, or in some places the necessary heat and light may be supplied in the form of electric energy, etc. In the last case this could be defined accurately, and could be compared according to the price per unit. But in the other cases mentioned accurate definition is not possible. In the case of fuels, perhaps it would suffice for virtual equivalents to be given equating (for heating purposes) the heating power, etc., of wood, lignite, and anthracite. In the case of tea, the grades of which are very numerous, and taste in regard to which is very variable, it is quite impracticable to attempt to base the comparison on physical standards. In such cases we may often ignore—without vitiating our general purpose of measuring the purchasing-power of money—the physical standard and include it on another basis, viz., that of preference or commonness of usage. Let us suppose that there are really n grades of tea on the market (the remarks will apply to all commodities of the same type) where n may be say 10, 20, or more, the use in any locality being—

$$(3) \dots q_1 \text{ of } g_1 \text{ at } p_1 + q_2 \text{ of } g_2 \text{ at } p_2 + q_3 \text{ of } g_3 \text{ at } p_3 + \dots + q_n \text{ of } g_n \text{ at } p_n$$

q denoting quantity, g grade, and p the price. If complete information existed it would be possible (though impracticable) to find the average price, and if this were taken in dealing with the particular item, when computing the cost of the composite-unit, it would be satisfactory. In all symmetrical distributions of statistical facts, however, the most frequent as well as the average case lies midway between the others: and in asymmetrical distributions—where the degree of asymmetry is constant—the average case occupies a definite and constant relation to the most frequent case: hence we are practically justified in adopting the *grade most commonly used* as if it were the only grade, and adopting its price. In any one locality probably the relation is very constant, and systematic error quite negligible. As between different localities the error may not be *wholly* negligible, though it will probably be very small, and much of the same order as other unavoidable limitations.

16. Composite-unit for special purposes.—Let us suppose that a unit is required for such a purpose as adjusting a *minimum* wage. The balance in this case between the items ought for obvious reasons to be exactly adjusted so that its efficiency as regards nutrition and general hygiene is satisfactory. The question of its economic possibility ought of course to be taken into consideration at the same time. It may also become necessary in times of difficulty to consider available alternatives among various commodities, especially foodstuffs, with a view to meeting shortages. Thus the appropriate composite-unit would be of a restricted character, limited to essentials, and susceptible of accurate identification.

On the other hand, a composite-unit, intended for measuring the purchasing-power of money generally, should be designed to include the largest possible number of *identifiable* commodities; these are taken as the gauge for measuring the fluctuations of the relation between money and commodities.

17. **Secular changes in the purchasing-power of money.**—In order to clearly apprehend the nature of the problem of tracing the secular changes in the purchasing-power of money, let us consider two such simple and important commodities as wheaten bread and sugar. The quality of the wheat from which the former is made has become more definitely fixed: the manufacture of flour improved and the whole process of bread-making such that a better and more definite article has resulted. The same is true of sugar. Canes and beets have been greatly improved, and refined sugar is an extraordinarily pure product. Thus, physically, bread and sugar are no longer quite the same commodity that they were 100 years ago. The price of a loaf of given weight, or of a given weight of sugar, may be taken as that of one of the items in the composite-unit; but the price is really for a *different grade of article*. A little consideration of analogous facts will show what any notion that a continuous relation can be made out is founded upon a misconception of the essential nature of the problem.

If we class "lighting" as a commodity, then we have to pass from rushlights to candles, to gas, to the incandescence of mantle lights, to electric lighting, and so on. If in original list rushlights were entered, the item disappears, to be replaced by candles, and so on, and finally by an electric unit. It is evident that we cannot merely enter the unit of lighting, *i.e.*, at one date a candle, and at a later date a unit of electricity. What we can do is, as the circumstances change, to include both in proportions, differing from year to year or decade till one passes out and the other takes its place. This is the nature of the change. Thus the price-index is changing its basis, slightly as regards a period of 1 year, but greatly as regards a period of 100 years or more. It is easy to see that its *validity* is unquestionable for tracing changes in the purchasing-power of money over short periods of time, and equally evident that comparisons over great periods are confused with other questions, *viz.*, the characteristic usages of mankind in regard to the various elements of life. One who studies the graph of a series of price-indexes and imagines that it discloses the real variations in the purchasing-power of money over the long period represented (as it does over short periods) *loses sight of the fact that the price-index does not mean the same thing for two points separated by long intervals of time*. Price-indexes are and can be only pseudo-continuous.

18. **Mode of ensuring pseudo-continuity.**—Let us suppose that, at a particular point of time, one of the periodic alterations of the composite-unit has become necessary; that is, we have discovered that we must revise the list of items in our composite-unit by introducing some new ones and varying the quantities of all or most of the remainder, perhaps even omitting some of them altogether. The result may be represented symbolically by such a scheme as the following, *viz.* :—

Old list, $U_c = a \text{ of } A + b \text{ of } B + c \text{ of } C + \dots + m \text{ of } M$

New list, $U'_c = \dots c' \text{ of } C + \dots + m' \text{ of } M + n \text{ of } N + p \text{ of } P$.

The first two commodities are omitted altogether: the quantities $c, d, \dots m$ become $c', d', \dots m'$ and new commodities are added, *viz.*, n of N and p of P , etc. Since the list is equally good whatever the magnitude of the items, *provided that their mutual ratios are not altered*, we can easily arrange that the price of U'_c (aggregate of cost of all the items in the new unit) is the same as that for U_c . Inasmuch as, when we find the cost of the new composite-unit U'_c for any later date, we then get the same result, whether we divide by the cost of U_c or U'_c , a list of the aggregates of expenditure for a series of years is made continuous, *i.e.*, in the same ratio as the price-indexes. Thus, suppose the change were made in the year 1911, we might have cost of aggregates as follows :—

Composite-unit 1.

Composite-unit 2.

Years	1908.	1909.	1910.	1911.	1911.	1912.	1913.	1914.
Aggregates = ..	201427	200884	205408	212014*	212014†	233316	234088	241668
Ratios $\times 10000 =$..	9501	9475	9688	10000	10000	11005	11041	11399

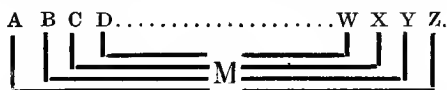
* Cost of U_c .

† Cost of U'_c .

It may suggest itself that, since the price-indexes are assuredly properly determined on the supposititious basis (which initially—at any rate—represents actual usage) for the years 1908 to 1911 (in the illustration above), and also for the years 1911 to 1914, they are therefore rigorously continuous. The invalidity of this view is seen from the following considerations:—If the composite-unit of any period had not ceased to represent exactly the usage of the community (or individual, class, people, etc.) *no change would have been necessary*. The fact that a change has to be made shews that the later values (as 1911 is approached in the illustration) cease to be exact, i.e., they are rigorously, nominal rather than actual: they would have applied had the usage remained constant. The continuity is not real: as the usage of the community changes it ceases to be valid. An illustration from the individual would again make the matter self-evident. Suppose a man in early life, aged 30, say, ascertained his usage and established a “composite-unit” for himself against the items of which he set out their prices. He thus commences his series of price-indexes. After 10 years he realises that his condition of life and habitual usage have so far changed that he deems it necessary to establish a new composite-unit. This now takes the place of the old one, and from 40 on to 50 he uses the new basis, correcting it each 10 years till 70, when he passes away. His son continues the work, joining on his evaluations of price-indexes, revising his list every 10 years; being followed in due course by his son; and so on. The heir is now in possession of *a continuous series of price-indexes*. At no point of time do they appear to be in any way faulty. Clearly, however, in course of time they can no longer be regarded as admitting of comparisons with past years, because the successive changes—despite the fact that each was relatively small—has so altered the basis of comparison that the aggregates of expenditure do not refer to the same things at all. Thus we see the sum of the divergence, each of which was perhaps slight, may be quite considerable in the aggregate, in which case it makes the earlier and later results non-comparable. They might, for example, not contain even a single commodity in common.

19. **The system of relations between money and commodities.**—In all questions of exchange-value, money is a common denominator. Excluding from consideration its dual or multiple functions (as a commodity) it is—as money—the one thing in which the entire system of exchange-relations between commodities is expressed. This may be represented as follows:—

Let A B Y Z denote general commodities, and M denote money, then the direct relation is always between M and A B Z. The indirect are A M B, A M C, etc., B M C, etc., which may be represented by the following diagram:—



It is clear from this alone, that if A to Z disappear, and are replaced by a to z, and later by a to z, it is meaningless to speak of change in purchasing-power of the one constant thing M for the relations with other things which originally existed and by means of which we measured its purchasing-power have vanished.

It is of course possible to formulate inquiries as to the relation which M bore to so much of each of A, B, ... etc., or a, b, ... etc., or a, b, ... etc., as were requisite to maintain life or to live according to some definable status, involving the use of these commodities, but the absurdity of any supposition (except upon some fictitious assumption of a purely arbitrary relationship) that a possible relation can be established through this, is self-evident. Money (M) is in constant relationship with commodities: thus its “purchasing-power” can be set out for any or all (in prescribed quantities) that exist. And hence comparisons can be made between those existing at different periods; as, for example, *in terms of gold*, so much of A, B, C, etc., would be equal to so much of a, b, c, etc., or of a, b, c, etc. In precisely the same way, if the variation in the value of money (its purchasing-power or reciprocal of the price-index) is to be determined in terms of commodities, they must continue to exist in the same unchanging form. Thus the exchange value of gold in terms of “wheat,” or of the “flesh of oxen,” or in terms of “copper,” “tin,” “silver,” etc., is *possible*. Consequently if only a *significant* permanent unit were available, we could measure the exchange value of gold in terms of this unit and hence, reciprocally, a common basis would be to hand for the measurement of the

purchasing-power of money. *There is, however, no satisfactory and significant unit other than gold available for such a purpose.* It is precisely because gold possesses certain valuable physical properties (viz., those of a noble metal valuable in the arts, etc.) that the consensus of practice has made it from time immemorial *the* commodity in which the *value* of others could be expressed. Even if a compound unit were established, such as that so much wheat, maize, oats, barley, etc., were to be regarded as a unit of value, the fluctuations of relation between the elements or constituents thereof would render its meaning equivocal, and make futile comparisons based thereupon. We thus see that the impossibility of obtaining price-indexes of unequivocal significance is a consequence of the nature of things, not merely of the arbitrary determinations or usages of the human race.

20. The effect of abnormal times upon Price-indexes.—Abnormal times greatly disturb the relationship between money and commodities in one of three ways, viz. :—

- (i.) By suddenly and greatly *cheapening* commodities, i.e., *increasing* the purchasing-power of money ;
- (ii.) By greatly *increasing* the price of commodities, i.e., *reducing* the purchasing-power of money.
- (iii.) By greatly *disturbing* the mutual price relations of commodities themselves, i.e., producing a *bouleversement* of the purchasing-power of money as among different classes of commodities.

The nature of these is easily seen from homely illustrations. When eggs, fruit, etc., are dear, their consumption falls off. In countries where there are famines in cereals (rye, rice, and so on), less is eaten and substitutes are used. If bacon, ham, poultry, cheese, etc., become very dear, people substitute cheaper foods. Thus the regimen on which the composite-unit was founded in normal times no longer applies to the actual conditions of the community. It is not unlikely that variations in food are of value hygienically, when they are restricted in amount, and the available regimens are adequate in quantity, and are agreeable. In the extreme case of famine, all satisfactory regimens become unavailable, and adequate quantities are not to be had. It may be mentioned in passing that in these extreme cases no manipulative control of the relations between wages, prices, and commodities is possible.

It is important to bear in mind that, *per se*, famine does not *necessarily* disturb the relations between commodities ; it may, for example, double, treble, quadruple or raise tenfold the prices of all, and thus raise the price-index in like measure. But so long as the *ratio of usage* in the several commodities is unchanged, the *constitution of the composite-unit is unaffected*. In short, famine or other abnormal situations do not *necessarily* vitiate the composite-unit, no matter to what extent the consumption of commodities is reduced. It is only when such conditions cause the *ratio of usage of the several items to differ materially from that adopted* that they can materially affect the validity of the adopted composite-unit. In fact, very considerable changes may take place in the composite-unit without affecting very greatly the price-index, because when the comparison is made the changed unit must be applied at both dates to measure the change in purchasing-power.

The question as to whether the changed unit is satisfactory from the standpoint of health, comfort, or taste, or not, is an independent question, and is one which does not directly affect the price-index question.

21. Effect of seasonal fluctuations in consumption of commodities.—There are many ways in which human needs change during the year. Fuel and lighting are a larger item in winter ; the fruit season produces some change in the nature of the foods consumed ; and certain commodities are dearer at one time than another, producing also a modification of the regimen. Thus even the perfectly ascertained constant composite-unit does not strictly represent human consumption of commodities except "on the average." Price-indexes, based upon a constant composite-unit, are therefore *not strictly accurate* for any particular date, though "on the average" they are accurate. The error, however, in nearly all cases is *quite* negligible.

22. For many questions expenditure is not the measure of importance of Commodities.—A composite-unit to form the basis for measurements of the purchasing-power of money should obviously be so constituted as to express the various essential needs of human beings in their appropriate proportion. No account

need be taken, in constituting this unit, of the magnitude of the expenditure on any particular item or series of items. On a superficial view, however, it would seem that it might well be taken into account. It is certainly plausible to assert that the various items must be allowed to be of an importance, proportionate to their ratio to the total. Thus the expenditure on bread and meats is respectively about $3\frac{1}{2}$ per cent. and 14 per cent. of the total expenditure: their relative significance is thus 0.035 and 0.140, or 1 is to 4. Thus we might say if these rise respectively 50 per cent. and 80 per cent. in price their relative significance would be only as 5.25 per cent. to 25.2 per cent., or as .0525 to 0.2520, or roughly as about 1 to nearly 5 (exactly 4.8).

We notice first of all a *practical* difficulty that would be introduced, viz., that the ratio of importance of every commodity is an ever *fluctuating quantity*, except where their prices move up or down in the same ratio. However, mere arithmetical difficulties ought not to operate against the introduction of sound methods: the mode of dealing with them could be easily attended to when the first principle was settled.

Suppose, then, there are three commodities, A, B and C, on which a given amount, say 140s., is spent in a given time: and we will suppose that the expenditure on them is 20s., 40s., 80s., respectively at date 1. Hence the importance of them is respectively $20/140$, $40/140$, and $80/140$, or $\frac{1}{7}$, $\frac{2}{7}$ and $\frac{4}{7}$. At date 2 their prices are found to have increased by respectively 50 per cent., 20 per cent. and 25 per cent. Thus they have become 30s., 48s., and 100s.; or 178s. in all, a rise, on the average, of about 27.14 per cent. Their relative importance on the hypothesis that it varies as the amount of expenditure is $30/178$, $48/178$, and $100/178$. The importance is best compared, say, when expressed as decimals. Thus for A, B and C respectively they are:—

(1) A, .14286; B, .28571; C, .57143. (2) A, .16854; B, .26966; C, .56180.

Are we to take the first series of values, of their relative importance; the second series; or some mean? Several means have been advocated, the arithmetic (A.M.); the harmonic (H.M.); the geometric (G.M.).* Without troubling about how these are found or why they are advocated they may be given, and are as follow:—

Arithmetic Means.			Geometric Means.			Harmonic Means.		
.15570	.27768	.56662	.15517	.27757	.56659	.15464	.27746	.56657
.15570	.27768	.56662	.15527	.27776	.56697	.15484	.27783	.56733

The lower line shews these numbers corrected by a common factor to make their sum unity. The possible geometric and harmonic means being less than unity without this correction, another mean (for which something may be said) is that obtained by taking the means of the numerators and the denominators in the first two series of fractions. This gives .15723; .27673; .56604 = 1.00000, and we shall denote it by N.D.M. Although these numbers do not differ greatly, the differences are not wholly negligible. Thus the weights expressing the degree of importance attached to the commodities are as follows:—

	Date 1.	A.M.	G.M.†	H.M.	N.D.M.	Date 2.
A	.14286	.15570	.15527	.15484	.15723	.16854
B	.28571	.27768	.27776	.27783	.27673	.26966
C	.57143	.56662	.56697	.56733	.56604	.56180
Sums	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

† The small "c" denotes that the means are corrected so as to give unity as their sum.

* These means between two quantities a and b are:—Arithmetic mean = $\frac{1}{2}(a+b)$; Geometric mean = $\sqrt{ab}$; Harmonic mean = $2ab/(a+b)$. Thus the means between 4 and 5 are respectively 4.5; 4.472136 and 4.444444.

For use in connection with the question of a minimum cost of living, it is obvious that the balance of a regimen should, if possible, not be disturbed. The items belong to the class necessities (*s*), not luxuries (*u*), see section 1. Hence we might suppose that on the whole the aggregate amount of each item rather than the balance between the items is likely to be disturbed. In any case the measure of the purchasing-power is unequivocal only on that supposition.

23. **The deduction of purchasing-power from the weighted ratios of increase in price.**—In the previous section we may suppose the prices to increase for commodities A, B and C respectively from 1 to 1.5, from 1 to 1.20, and from 1 to 1.25. Let us now compare the effect of the aggregate expenditure (or composite-unit) method with the idea of weighting. The aggregate cost of the same quantity of commodities was (as already stated) first 140s. (at date 1), and then 178s. (at date 2): thus the price-index rose from 1000.00, say, at date 1, to 1271.43, say, at date 2; and this result is unequivocal. Thus we have—

Basis ..	Correct	U_1 , Date 1.	A.M.	G.M.	H.M.	N.D.M.	U_2 , Date 2.
Index ..	1271.43	1000.00	1275.04	1274.92	1274.82	1275.47	1278.65

Thus, though it seems plausible that the price-index should be based upon the method of weighting the items, according to the magnitude of the expenditure upon them, only the first result is correct, viz., that based upon the ratio of expenditure at the date from which the ratio of increase is estimated.

It has already been pointed out that a particular treatment may lead to an arithmetical process which reproduces the computation of an apparently different method (the two methods, however, being really the same). Here also the procedure in a roundabout way merely results in the division of the final cost of the constant regimen by the initial cost, and thus obviously is sound only when the composite-unit is unchanged.

24. **Recapitulation of the nature of the problem of ascertaining the purchasing-power of money.**—When, at a given point of time, any unit of money is exchanged for a definite quantity of any commodity, of a particular grade, this quantity is the expression of the purchasing-power of the money-unit (i.e., in relation to the commodity in question). In like manner the purchasing-power can be expressed in terms not of its exchange-value in respect of one commodity, but in respect of definite quantities of a series of commodities. The number of ways in which such a series can be formulated is obviously without limit, and therefore the question of relating purchasing-power becomes definite only when both the commodities and the quantities and grades thereof are specifically fixed. Practically it is necessary that they should readily be identifiable both as to character and quality or grade. A list of commodities with the grades and quantities required by any individual, or by a group of persons, may be called a *composite-unit* and may be used as a qualitative and quantitative common-denominator for the measurement of the changing relation between money and the selected group of commodities.

With different schemes of expenditure (or composite-units) money has different purchasing-powers even for an individual. The most significant of these is that purchasing-power, which is based upon a composite-unit in accord with the actual usage of the individual; similarly for any group or class of individuals, whether small or large, if the composite-unit represent their usage on the average, it may be regarded as applying to the total group, since the average usage may be attributed to each member. We can make any date at which prices are obtained a *reference date*, and computing the aggregate-cost of the composite-unit at that date, we can regard the total amount as a *unit of expenditure*. If, then, the cost of the same composite-unit be made out to any other date, this cost, divided by the original cost, is the price-index of the second date in regard to the first, and is the reciprocal of the purchasing-power, since it shows the amount that has to be paid for the composite-unit on the second occasion, that on the original occasion being regarded as 1. In order to avoid the use of decimals, however, it is convenient to express the unit on the original date (to which all others are referred) as 100, 1000, 10,000, etc., according to the precision with which it is desired to shew the results.

Inasmuch as human affairs change, the composite-unit used for this purpose cannot be regarded as permanently applicable: it has to be amended from time to time. Consequently, although price-indexes are apparently continuous, they are not really so: when taken over long periods of time they do not refer to the same physical unit of reference.

For mere arithmetical convenience it is well, when changing from an old to any new series of commodities, to arrange that the new composite-unit shall cost the same as the old on the date of change. It is not necessary to do this, but, if done, the aggregates can then be used continuously, the ratio of one aggregate-cost to another shewing the relative purchasing-powers or price-indexes as the case may be.

If at each date when the list is changed, *i.e.*, when the composite-unit is altered, the change be insignificant, the result is more nearly continuous; whereas if the change be great the result is really more markedly discontinuous. It is obvious that the change can be made as rarely or as frequently as we desire. Human customs, however, are ordinarily sufficiently constant to admit of a change being made not more frequently than say every five or ten years. Towards the end of each period, perhaps the composite-unit will ordinarily have ceased to accurately represent the usage of the individual or community, concerned in the change in the purchasing-power, but no very distinct advantage would be gained by annual changes of the unit; indeed, in the case of the newly introduced commodities, prices may not be available in the past. So long as the composite-unit fairly well represents the usage of the community, it reflects the purchasing-power of money with great accuracy, *i.e.*, that purchasing-power which is significant to the community in question. The method of determining price-indexes or purchasing-power by means of the cost of the composite-unit is one which not only is theoretically unimpeachable: it is also in practice by far the most simple method. It has also the advantage that one can ascertain instantly, and in the simplest possible way, the exact effect of any uncertainty in the data. Other methods are not only theoretically less satisfactory, but also obscure the intrinsic nature of the method adopted.

PART III.

TECHNIQUE OF COMPUTING PRICE INDEXES.

SYNOPSIS.

1. Essentials of problem.
2. The purpose of price-indexes.
3. Price-indexes for deducing the quantity-ratios from the values of imports.
4. The composite-unit for price-indexes of imports.
5. Price-indexes for relating quantities to values of exports.
6. Ascertaining the elements of a composite-unit.
7. How far may composite-units formulated for a particular purpose be used generally ?
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9. Price-indexes must be reversible.
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12. Computation of price-indexes when quantities used are identical at both dates.
13. Computation of price-indexes when quantities used are *not* identical at both dates.
14. The disadvantages of the price-ratio methods.
15. Practical difficulties in obtaining accurate price-indexes.
16. Character of items in composite-unit.
17. Changes in composite-unit.
18. Revision of price-indexes to secure high accuracy of long periods.
19. Price-indexes with a periodic change of the composite-unit.
20. Omission of items from composite-unit.
21. Variations of price-levels.
22. Advantages of a price-index over a price-level.
23. On the discontinuity of price-indexes.
24. Substitution of equivalent items in a composite-unit.

1. **Essentials of problem.**—Among commodities of ordinary usage are the following, viz. :—

- (i.) Those which can be readily specified and identified as regards essential character, and quality or grade.
- (ii.) Those which, though not readily specifiable and identifiable as regards grade and physical qualities, etc., can, nevertheless, be definitely determinable by common usage, and may be regarded as equivalent.
- (iii.) Those which are not determinable because of difficulties of specification and recognition as between one date and another, or one locality and another.
- (iv.) Those whose distribution and use are infrequent or in any way not characteristic of sufficiently large groups to have significance in the formulation of a composite-unit.
- (v.) Those which, though embraced in (i.) to (iv.) are used in such small quantities as to be negligible in any general considerations.

Bearing in mind that the essence of any accurate evaluation of a price-index, is that like shall be compared in price with like, it is obvious that in the formulation of the composite-unit only (i.) and (ii.) can be included. To attempt to introduce (iii.) and (iv.) would have the effect (a) of impairing the accuracy of the index without materially extending its significance, and (b) of *confusing change of price with change of standard*. The notion that price-ratios might be in some way combined with the result so as to extend its significance is founded upon a misconception of the whole technique of ascertaining a price-index. If the composite-unit is too restricted, the best way (ordinarily) of extending it is to include other items, with all their measure of uncertainty. The result will then be quite definite, provided we know the measure of the uncertainty, for we can in such a case very readily ascertain what effect the uncertain items in the composite-unit have upon the aggregate. Thus let us suppose that for the year 1916 the aggregate expenditure was 21025 ± 17 ; and that for the year 1917 was 23059 ± 48 . In such a case the uncertainty in the price-index ranged between $23107 \div 21008$ and $23011 \div 21042$. Hence we have the results :— Minimum, 1093.6; Medium, 1096.7; Maximum, 1099.9. These results can be expressed $21025 (1 \pm 0.00081)$ and $23059 (1 \pm 0.00208)$, which enables us—seeing the quantities are small—to write down at once 1096.7 ($1 \pm .00289$). Having the result based upon the certain items before us we can at once see whether the somewhat uncertain items should or should not be included, for the value of the price-index lies between 1093.6 and 1099.9. If on omitting the uncertain items the index lies outside these limits, it is clearly preferable *not* to omit them. We shall later consider the case where every element is regarded as subject to some measure of uncertainty.

The essentials then for accurately finding price-indexes are the following :—

- (a) The formulation, subject to the limitations just indicated, of a composite-unit, shewing the commodities to be included and the quantity (average usage) of each.
- (b) The obtaining of the prices of these commodities (either the modal, *i.e.*, the most frequent, or the average prices).
- (c) The computation of the total cost of the composite-unit by attributing the price to each item according to its quantity, and forming the sum of the items.
- (d) The obtaining of the price-indexes (by divisions) from the cost, at different times or places, of a definitely constituted composite-unit, some particular year and place being constituted a time and place of reference.*

Of these (a) is determined by the *purpose of the index*. We shall deal with several principal purposes in the next section. Afterwards we shall consider the finding of the prices, and the methods of computation for the purpose of arriving at the price-indexes.

2. The purpose of price-indexes.—There are several purposes served by means of price-indexes. In the most general view their purpose is to ascertain the exchange-relation between the unit of money (gold) and the totality of other commodities in the quantities in which the latter occur, or are used : see section 19 of the preceding part. The solution of this problem is, however, impracticable because of its magnitude. The most general practicable solution is that in which all the *most significant commodities* are selected and are assumed to represent the whole.

The more restricted purposes then are as follow :—

- (i.) To find the general exchange-relation between money and all important commodities in the proportion of their usage.
- (ii.) To find the general exchange-relation between money and the commodities used by a particular State, or class within the State.
- (iii.) To find the general exchange-relation between money and the commodities imported into a country (a) for the purposes of analysing the volume of trade, or (b) generally.
- (iv.) To find the general exchange-relation between money and the goods exported from a country (a) for any special purpose, or (b) generally.
- (v.) To measure the purchasing-efficiency of wages and salaries for the classes receiving them.
- (vi.) To shew the possible differences between payment in kind and in money in regard to services, contracts, annuities, etc.

* The index for this basic year and place is usually made 100, or 1000. If we want high precision for particular comparisons it might even be made 10,000.

These embrace the main usages, and, it will be seen, must be taken into account in establishing the technique of ascertaining the appropriate price-index. We shall deal first with the more restricted use in the case of imports, and exports.

3. Price-indexes for deducing the quantity-ratios from the values of imports.—

If external trade, either imports or exports, be measured solely by value, it is of course impossible to say whether its volume has actually increased or not. For if the *quantity* of goods were identical for any two years, and higher *values* were attributed to them for the later year than were attributed on the former, the imports would *appear* to have increased, whereas in fact only their price had increased. Thus to decide certain obvious economic questions both quantity and value are required in trade-returns. If from the value of the imports we wish to compare the magnitude of the imports with that of other years, we must reduce the records of value by means of an appropriate price-index. For suppose the value of imports had increased from 100 millions to 120 millions, it would denote an increase in volume of 20 per cent. if the price were unchanged and no increase in volume whatever if the prices had all advanced 20 per cent.

Suppose the price-indexes were 1000 and 1100 and the increase in the value of the total trade was 20 per cent., i.e., from 100 to 120; then half of this difference would represent the effect of price: that is, in the second year the same quantity of goods as in the first year cost 110 ($100 \times 1100 \div 1000$). Thus the real increase was in the ratio of 110 to 120, or as 100 to 109.091; that is, the quantity had increased not 20 per cent., but 9.091 per cent. (The *value of this increase* expressed as a percentage of the total value of imports in the first year is of course also 10 per cent.)

We see then that if i denote the price-index for *imports*, and I their total *value*, the ratios of their quantities, Q , are:—

$$(4) \dots Q_1 : Q_2 : Q_3 : \text{etc.} = \frac{I_1}{i_1} : \frac{I_2}{i_2} : \frac{I_3}{i_3} : \text{etc.}$$

the suffixes denoting the successive years. Or we might express this relation—

$$(5) \dots Q_1 i_1 : Q_2 i_2 : \dots Q_n i_n = I_1 : I_2 : \dots I_n.$$

The ratio Q_1/Q_2 in this sense is a sort of *general expression* for the ratio of the quantities of the aggregates of trade.

It is convenient to have an index for the purpose of representing the volume of trade in succeeding years, viz., the value of the ratio $Q/Q = q_{n1}$ say, so that q_{n1} will denote what may be called the ratio of the *generalised quantities* of the trade at date n compared with date 1. Thus from (4) we have—

$$(6) \dots q_{n1} = (I_n/i_n) \div (I_1/i_1) = \frac{i_1}{i_n} \cdot \frac{I_n}{I_1}$$

These ratios could be multiplied by 1000, 10,000, etc., the accuracy depending on that to which the price-indexes were expressed.

4. The composite-unit for price-indexes of imports.—

The composite-unit for dealing with questions relating to imports must, of course, be formulated from the imports themselves by selecting such commodities as may be identified for at least two succeeding years, the general principle of identification, etc., being substantially that already indicated in section 1.

In obtaining price-indexes for imports, however, it is not always possible, and for certain purposes it is not even specially desirable, to avoid indifference as regards grades and qualities. This becomes obvious from the following considerations. Suppose equal quantities (by weight or measure) were imported in successive years (though this is not essential), the grade being the same but the price 10 per cent. higher in the second year. In such a case the price indexes would be in the ratio 1000 and 1100, and by dividing the values by these numbers we should have the quantities expressed in their true ratio: that is, in the case supposed they would be equal. It is clearly immaterial from the standpoint of quantity merely whether the advance in price is due to improvement in quality or not: consequently, if there be no reason for recognising (as however there would be, in questions of standard-of-living), the difference of grade, such differences may be ignored.

In national economics, questions of import and export are concerned largely with the *volume of trade*. This is often the element of chief moment. We may, therefore, unhesitatingly use price-indexes obtained from data in which there has been no nice discrimination in regard to grade, provided that fact is duly recorded, and provided also that the price-indexes so ascertained are not otherwise misapplied—as they would be, for example, in any attempt to use them in criticising the significance of advances in price for wage-earners.

We see that the *purpose* of a price-index affects the technique of its computation. Thus if, in the case of imports, the price-index is to be applied to any questions in which the grade of goods used is a material feature, that computed for deducing a general measure of quantity from value is *inappropriate*, and its use would lead to erroneous results. And inasmuch as for so many commodities the standardising of qualities or grades over long periods of time is practically impossible, or at least very difficult, we can hope to make accurate deductions only for points of time relatively near together.

5. **Price-indexes for relating quantities to values of exports.**—Precisely as the values of imports fail to shew the volume of trade, so also the value of exports, E , do not shew the volume of exports. Hence in the same way an appropriate price-index, e , must be applied; so that we shall have, as in the last section—

$$(7) \dots Q'_1 : Q'_2 : \dots Q'_n = E_1 / e_1 : E_2 / e_2 : \dots : E_n / e_n$$

Q' denoting what may be called the "generalised quantity" of the export. We may call the ratios of the volume of trade the *index of the volume of trade*, and denote it by q' , then we shall have as before—

$$(8) \dots q'_{n1} = (E_n / e_n) \div (E_1 / e_1) = (e_1 / e_n) (E_n / E_1).$$

By the use of export price-indexes, therefore, we can analyse the volume of the exports just as we could that of the imports.

The essence of the methods for determining both price-indexes, viz., the indexes for imports and exports, is that the usage of commodities must be (substantially) the same for the two dates to be compared (i.e., in either case) if the result is to be unequivocal. If the composite-unit has varied in either case the results cannot be strictly comparable, and the application of corrections to values in order to obtain the generalised quantities is then only approximate.

We shall deal later with a particular type of price-index which we shall call *price-levels*.

Where exports (or imports) have a well-defined periodic fluctuation the composite-unit should be the mean of a whole fluctuation period: this may be taken to be sufficiently representative of the entire range. It will be seen later (viz., in the section on price-levels), that the mean of several years gives very uniform results.

6. **Ascertaining elements of a composite-unit.**—Two fundamentally different methods are available for ascertaining the constitution of the composite-unit, viz.:

(a) Compilation from household budgets, etc.

(b) Deduction from the total consumption of a population.

When employing method (a) the group of households should be so constituted as to give good average results: hence the frequency of those differently constituted as to numbers and usage of commodities should at least roughly conform to the class in the community selected, or for the entire community, as the case may be. The method (b) is the most general, and is probably the safest: the unit so computed represents the usage of the community as a whole.

Where both methods are practicable, the advantage of using both and of taking the means (duly weighted according to the probability of their accuracy) is self-evident.

The following values for the item-units were found for the constituents of the composite-unit for Australia generally and for Western Australia, and are given as an illustration of the type of composite-unit required in inquiries relating to the *varying purchasing-power of money in regard to food*. These regimens or composite-units have been found in quite different ways. The first was determined by method (b), and is based upon an estimate of the consumption in Australia carried over about 10 years. It thus represents the usage of the entire community. The second was determined by method (a), and represents the usage of workers in Western Australia, mainly Perth, observed over a short period of time, about 3 months.

Composite-unit Representing Weekly Consumption of a Family of about 5 Persons.

COMMODITY.	AMOUNT.		COMMODITY.	AMOUNT.	
	C'wealth.	W. Aust.		C'wealth.	W. Aust.
	lbs.	lbs.		lbs.	lbs.
Bread ..	21.1	19.87	Potatoes ..	20.2	11.3
Flour ..	6.7	5.5	Onions ..	1.5	1.4
Tea ..	0.7	0.7	Milk ..	17.6*	12.2†
Coffee ..	0.05	0.07	Butter ..	2.1	2.4
Sugar ..	10.4	6.4	Cheese ..	0.3	0.3
Rice ..	1.1	0.8	Eggs ..	0.8‡	3.39§
Sago ..	0.2	0.2	Bacon ..	0.8	0.4
Jam ..	1.6	2.5	Ham ..	0.2	0.1
Oatmeal..	0.8	1.0	Beef ..	8.7	5.4
Raisins ..	0.3	0.7	Mutton ..	7.5	3.9
Currants	0.3		Pork ..	0.8	0.8

* 6.8 quarts. † 4.7 quarts. ‡ 4.9 eggs. § 21 eggs.

These quantities may be called the *mass-units* of the regimen (or of the composite-unit).

In view of the fact that these composite-units are actually deduced from the best evidence available, they afford an interesting test of the variation of price-index determinations based upon a general composite-unit, and one representing only a particular class. We take for this purpose (at random) the Commonwealth averaged prices for the years 1912 and 1917, and on these bases the *total-expenditures** would be

Commonwealth basis 1912 = £313,907*, 1917 = £395,942*
W. Australian basis 1912 = £268,732*, 1917 = £325,257*

Thus the price-index for the former year being 1000.0 for both the Commonwealth and Western Australian bases, the price-indexes are respectively 1261.3 and 1210.3. We are able to see from this that *quite considerable differences in the regimens* (or composite-units) *do not very greatly affect the price-indexes*: although they may, of course, considerably affect the aggregates of expenditure. Meticulous accuracy is, therefore, not necessary in regard to composite-unit.

The composite-unit furnished in the preceding table is of course specially suitable for determining the variations in the purchasing-power of money, in relation particularly to food and groceries and to other things also in so far as they reflect the general variation in the value of money. To that extent they may be applied generally.

7. How far may composite-units formulated for a particular purpose be used generally?—Owing to the very large number of commodities the *standard* or *quality* of which cannot be identified, it is not unfrequently necessary to exclude them from the composite-unit constituting the basis of comparisons. The nature of the criteria for this has already been considered, viz., in sections 13 to 15 of Part I. The principle to be followed is as follows:—

- (i.) If the grade of a commodity be so changed that the price must be affected thereby, to include it would be to vitiate the price-index by *change of standard* (e.g., if a better article is dearer in price it does not shew that the purchasing-power of money has declined).
- (ii.) If what is virtually a new commodity nominally takes the place of an old one, it ought—in general—to be excluded in ascertaining a price-index; because the difference in price is not change in the purchasing-power of money. (This is analogous to the preceding case).
- (iii.) If the common-elements of usage at two dates or at two localities be restricted to a given number of commodities the only unequivocal comparison is that which is made on this number. This, however, may be of very limited value.

* For a considerable unit of time or for a large number of persons. The time or number is material. They, however, are the same in each case.

If we are justified in assuming that the prices (or cost) of the balance of the commodity-usage (or other usage) has, notwithstanding all variations between one item and another, increased (or diminished) in the same proportion, then the composite-unit considered is of wider application than its immediate purpose. Thus *price-indexes based upon a limited number of commodities are often properly regarded as of general application*, unless it be known that the application ought to be restricted, that is unless it be known that the omitted particulars (assumed of course to be of the same standard) differ on the average from what is indicated by the price-index.

This matter will be more fully considered hereinafter.

8. Price-indexes ascertained from price-ratios.—A price-ratio is the ratio of the price of a commodity at one given date compared with its price at another given date. *It is thus independent of quantities or units of measurement*; it matters not whether the price is per pound, per ton, per yard, per article, etc.

Let p, q, r , etc., be a series of price-ratios for commodities P, Q, R , etc.; and let us suppose that the quantities of the mass-units, or the quantities used are a, β, γ , etc., respectively. We shall shew that *the price-index can be fairly accurately computed from the price-ratios, provided certain safeguards are attended to*. Considering primarily two items only, let us suppose that at date 1 the price of $P = 3$, and of $Q = 4$; and at date 2 the prices were 6 and 5 respectively.* Then the price-ratios P and q are as follow, viz.:—

$$P = 6 \div 3 = 2 \text{ and } q = 5 \div 4 = 1.25.$$

If we do not know what influence (weight) each should have on the result for the two combined, we should know that, in so far as these two commodities are concerned, the price-index must have been between 2000 and 1250, the mean being 1625, the reciprocal of which gives 615.3846, the date of reference being the first date. Suppose the second date is made the date of reference, then we should have $p' = 3 \div 6 = 0.5$ and $q' = 4 \div 5 = 0.8$. Thus it would appear that the price-index lies between 500 and 800; the mean being 650, not 615.3846 as before. The reciprocal index is 1538.4615. If we suppose that the usage was one unit of each, we should have for the price-index $(6 + 5) \div (3 + 4) = 1\frac{1}{3}$, or for the second date as reference date $(3 + 4) \div (6 + 5) = \frac{7}{11}$; that is the price-index for the two commodities combined was 1571.4286 or reciprocally 636.3636, either number being (when accurately expressed) *exactly* the reciprocal of the other. Thus we have three different results, according to which way we calculate, viz.:—

1625.0; 1538.46; 1571.43 or 615.38; 650.00; 636.36.

We see, therefore, that only one of the methods indicated is *reversible*: that is only one of the methods gives identical results whichever date we make our basis.

9. Price-indexes must be reversible.—Since it is quite arbitrary whether we adopt one date or another as a date of reference, by whatever arithmetical (or other) method we deduce a price-index it must be consistent with the price-index deduced with any other date as basis. Thus if the two dates are say 1910, and 1915 and the price-index for 1910 is 1000, and for 1915 is 1500, it means that £1500 is required to buy in 1915 what £1000 would buy in 1910. This is the same thing as saying that £1000 (or $\frac{2}{3} \times 1500$) is necessary to buy in 1915 what could have been bought for £666 $\frac{2}{3}$ (or £666 13s. 4d.) in 1910. Thus if the price-index for 1915 is made 1000, that for 1910 is 666.6667. We see, therefore, that if we base the indexes on the *total of the expenditures* (or aggregate cost) of the composite-unit, this condition must always be fulfilled, but it was not fulfilled in the other cases. We cannot therefore regard arithmetical means as satisfactory, plausible as it might seem to do so.

Let us still suppose that the quantities of commodities P and Q were one unit of each: thus we have for the price-index for the year 1910 (as the basic year) $P = 2$; $q = 1.25$. The *geometric mean* of these is $\sqrt{(2 \times 1.25)} = 1.5811388$, the reciprocal of which is 0.6324553. If 1915 is made the basic year we have $p' = 0.5$ and $q' = 0.8$. The *geometric mean* is $\sqrt{(0.5 \times 0.8)} = 0.6324553$ as before. Hence

* The particular unit is of no moment.

the geometric mean also possesses the property of being reversible, and it is the only mean which has that property. More generally we have for the geometric mean of a series of quantities :—

$$(9) \dots\dots \frac{1}{\sqrt{(p \cdot q \cdot r \dots\dots \text{etc})}} = \sqrt{\left(\frac{1}{p} \cdot \frac{1}{q} \cdot \frac{1}{r} \dots\dots \text{etc.}\right)}.$$

as is obvious algebraically. We note, however, that the above result, 1.5811388, does not agree with any of the three preceding values. Thus, although satisfactory in so far as reversibility is concerned, *it is not correct if the aggregate-cost (or expenditure) method is correct, as we have shewn it is.*

10. In price-indexes reversibility is a necessary but not a sufficient condition.—Any satisfactory method of calculating a price-index must have the property of reversibility: that is a test condition. Suppose for example that, starting with the year 1910 as base, we make the price-index 1000, and using a particular method of calculation, we find that the price-index for the year 1915 is 1600. The ratio of 1000 to 1600 is the same as the ratio of 625 to 1000, hence if we had made the base 1000 for the year 1915, the price-index of the year 1910 must be 625. If, therefore, any adopted method of calculating price-indexes does not give this result, i.e., automatically fulfil this requirement, it is fundamentally a wrong method, and should be excluded as an inappropriate method of computation. This leaves available two satisfactory methods only, viz., the method of comparing the aggregate cost of composite-units, and the method of finding the *geometric means* of suitably-weighted price-ratios. We shall see that the weighting for a price-ratio is in the form of a *power* to which the price-index must be raised, and shall now shew that the assumption of identical quantities is an essential, though of course the quantities of the commodities actually used may not be the same on both occasions. This, however, does not effect the question of reversibility.

11. Weights of price-ratios must be means of relative expenditures on compared dates.—It is a plausible proposition that the economic significance of a commodity should be measured by the amount of expenditure thereon. Thus, if, on the average, a community spends three times as much money on meat as it does on bread, the economic importance of meat may be regarded as three times that of bread. We have already shewn, however, that this leads to a variable relation, due to want of uniformity in the changes of price. We are, therefore, on the horns of a dilemma; either the expenditure must be regarded as constant, or the usage must be regarded as constant.

If we made expenditure constant the question of the purchasing-power of money would cease to have any significance. At the most, therefore, we must suppose that there are only relatively small variations in the quantities used, and, as already shewn, an unequivocal meaning to purchasing-power exists only when the quantities of the commodities are assumed to be identical on the two dates. If, therefore, the actual quantitative usage is different on the two dates, we must adopt some mean (arithmetic or geometric, etc.) between the two, in order to establish an unequivocal basis of comparison, otherwise the result deduced would be the combined effect of change in purchasing-power with change of standard-of-living, or of usage. We must assume, therefore, that the mean quantities are the quantities used on each of the two occasions, and that the weights to be attributed to the price-ratios have some value lying between the relative expenditure on the commodities at the two dates compared. We shall shew later, by arithmetical examples, that the best mean is the geometric, but we shall also shew that the arithmetic mean gives nearly the same result, and is sufficiently accurate.

12. Computation of price-indexes when quantities used are identical at both dates.—Let us now, in the case illustrated in section 8, suppose that the cost of P and Q was expressed in shillings, say 3s. and 4s. respectively, on the first date, and 6s. and 5s. respectively on the second date. Thus the proportionate cost of each commodity was $\frac{3}{4}$ and $\frac{4}{5}$, respectively, on the first date, and $\frac{6}{5}$ and $\frac{5}{4}$, respectively, on the second date. If we make the weight dependent on the *relative expenditure* on

the two items we have, therefore, a different ratio on the two occasions, viz., for P, $\frac{3}{7}$ and $\frac{6}{11}$, and for Q, $\frac{4}{7}$ and $\frac{5}{11}$. For the purpose of examining their influence on the result we shall take both the arithmetic and geometric mean of these quantities. Thus we have :—

Commodity Arithmetic Mean.**Geometric Mean.**

$$\text{For P ; } \frac{1}{2} \left(\frac{3}{7} + \frac{6}{11} \right) = \frac{75}{154} = 0.4870130 ; \sqrt{\left(\frac{3}{7} \cdot \frac{6}{11} \right)} = \sqrt{\frac{18}{77}} = 0.4834938$$

$$\text{For Q ; } \frac{1}{2} \left(\frac{4}{7} + \frac{5}{11} \right) = \frac{79}{154} = 0.5129870 ; \sqrt{\left(\frac{4}{7} \cdot \frac{5}{11} \right)} = \sqrt{\frac{20}{77}} = 0.5096472$$

The sum of the arithmetic means is, of course, unity, but that of the geometric means is not unity. Suppose now that we regard these as the weights of the two price-indexes, and denote them by u and v , and by u' and v' respectively. First of all we take the arithmetic mean weights u and v ; we then have :—

$$(10) \dots \log \{ (p^u \cdot q^v)^{\frac{1}{u+v}} \} = \frac{\frac{75}{154} \log 2 + \frac{79}{154} \log 1.25}{\frac{75}{154} + \frac{79}{154}} = .19631997$$

$$= \log. \text{ of } 1.571520.$$

This multiplied by 1000 is 1571.520, which corresponds almost exactly with what was found by comparing the aggregate costs, viz., 1571.429 (expressed to three decimal places). If we used the values of u' and v' , viz., .4834938 and .5096472, the sum of which is .9931410 we should get :—

$$\frac{u' \log 2 + v' \log 1.25}{u' + v'} = \frac{.19493605}{.9931410} = .19628235 = \log \text{ of } 1.5713839.$$

This multiplied by 1000 is 1571.384, which is still nearer the result obtained, by comparing the aggregate cost, the three results being respectively :—

Aggregate cost result = 1571.429. A.M. weights result = 1571.520.
G.M. weights result = 1571.384 ;

A.M. denoting arithmetic mean, and G.M. geometric mean. These results are thus seen to be sensibly identical.

13. Computation of price-indexes when quantities used are not identical at both dates.—Let us take another elementary example, and suppose that 10 units of commodity P at 2s. each and 6 units of commodity Q at 5s. each are used at date 1, and 5 units of P at 6s. 6d. and 4 units of Q at 7s. are used at date 2. Thus the expenditures are :—

At date 1.

$$P \dots 10 @ 2s. = 20s. ; \quad Q \dots 6 @ 5s. = 30s. ;$$

At date 2.

$$P \dots 5 @ 6s. 6d. = 32s. 6d. ; \quad Q \dots 4 @ 7s. = 28s.$$

The aggregates, therefore, are 50s. and 60s. 6d., their ratio being as 1000 : 1210. This, of course, is *not a price-index*, because the quantities are different. If, in order to make a comparison of expenditures, we take, as the regimen, the means of the numbers of units, we have :—

$$\frac{1}{2} (10 + 5) = 7\frac{1}{2} \text{ of P ; and } \frac{1}{2} (6 + 4) = 5 \text{ of Q.}$$

From these we obtain the following results, viz. :—

Cost at Date 1.	Cost at Date 2.	Ratio of Costs.
$(7\frac{1}{2} \times 2s.) + (5 \times 5s.) = 40s. :$	$(7\frac{1}{2} \times 6s. 6d.) + (5 \times 7s.) = 83s. 9d.$	$1 : 2.09375$

Thus the aggregate-cost method gives a price-index of 2093.75, that at date 1 being 1000. Working, however, by price-ratios we have for P the price-ratio = $3.3 = p$; for Q the price-ratio = $1.4 = q$. The *relative expenditures* for P and Q respectively

were for date 1, $\frac{20}{50}$ and $\frac{30}{50}$, or $\frac{2}{5}$ and $\frac{3}{5}$; and for date 2, $\frac{32.5}{60.5}$ and $\frac{28.0}{60.5}$, or $\frac{65}{121}$ and $\frac{56}{121}$

Thus—

$$u = \frac{1}{2} \left(\frac{2}{5} + \frac{65}{121} \right); v = \frac{1}{2} \left(\frac{3}{5} + \frac{56}{121} \right); \text{ or } u' = \sqrt{\left(\frac{2}{5} \cdot \frac{65}{121} \right)}; v' = \sqrt{\left(\frac{3}{5} \cdot \frac{56}{121} \right)}$$

that is, $u = 0.468595$; $v = 0.531405$; or $u' = 0.463547$; $v' = 0.524105$. Using first one mean and then the other and noting that in the first case $u + v = 1$, we have log ratio required = $.468595 \log. 3.3 + .531405 \log. 1.4$ or

$$= (.463547 \log. 3.3 + .524105 \log. 1.4) \div 0.987652.$$

The two results are 0.3206262 and 0.3209045, which are the anti-logarithms of 2.09231 and 2.09365. Multiplying these by 1000, we have for the price-indexes obtained in the several ways:—

Aggregate-cost result = 2093.75 : A.M. weights result 2092.31 : G.M. weights result 2093.65.

As before, we see that the result when we use weights, which depend on the geometric means of the relative expenditures, are of higher precision than the result obtained when we use those which depend upon their arithmetic means. But we see also that even the arithmetic means give a very accurate result. The case taken is a very severe test, and yet the results are practically identical. *Price-indexes, therefore, can be found from price-ratios*, provided we properly weight the individual ratios: to secure high accuracy these weights must be the arithmetic means (or preferably the geometric means) of the relative expenditures on the two occasions—of a common unit of usage, viz., the arithmetic mean of the units used on the two occasions, if they are not identical.

14. **The disadvantages of the price-ratio method.**—We see that the price-ratio method when accurately used, is a very complicated one,* whereas the aggregate-expenditure method is quite simple. When the price-ratio method is applied to a large number of cases it becomes so tedious, as to be quite impracticable, and moreover, the large amount of arithmetical work involved is both useless and avoidable.

Since any accurate determination of a price-index, or of its reciprocal, viz., the purchasing power of a money-unit, requires that the regimen shall be predetermined (whether we use price-ratios or not), it is much more simple, and is more accurate to apply the prices to the items constituting the composite-unit, and thus find the aggregate-cost for each of the dates. The quantities of these items to be taken into consideration may be called the *mass units of the regimen*; or the *elements of the composite-unit*. Let the prices at dates 1 and 2, be $a_1 = a(1-x)$ and $a_2 = a(1+x)$, $b_1 = b(1-y)$ and $b_2 = b(1+y)$, and so on. Then we have:—

$$(11) \dots p \equiv \frac{a(1+x)}{a(1-x)} = \frac{1+x}{1-x}; \quad q \equiv \frac{b(1+y)}{b(1-y)} = \frac{1+y}{1-y}; \text{ etc.}$$

We can then shew that the aggregate-expenditures method gives for the logarithm of the price-index, I ,

$$(12) \dots \log I = 2 \left\{ \frac{aax + \beta by + \dots}{aa + \beta b + \dots} + \frac{1}{3} \left(\frac{aax + \beta by + \dots}{aa + \beta b + \dots} \right)^3 + \frac{1}{5} \left(\frac{aax + \beta by + \dots}{aa + \beta b + \dots} \right)^5 + \dots \right\}$$

* The controversy between Jevons and Laspeyres (see Labour Report No. 1, already referred to, appendix pp. xxxv., xxxvi.) shews how easily it is to fall into error in regard to the significance of the matter.

The method of price-ratios, with arithmetic means of the relative expenditures for the same mass-units, gives, however, for the logarithm of the price-index J :—

$$(13) \dots \log J = 2 \left\{ \frac{aax + \beta by + \dots}{aa + \beta b + \dots} + \frac{1}{3} \left(\frac{aax^3 + \beta by^3 + \dots}{aa + \beta b + \dots} \right) + \frac{1}{5} \left(\frac{aax^5 + \beta by^5 + \dots}{aa + \beta b + \dots} \right) + \dots \right\}$$

The difference between the two is :—

$$(14) \dots \log I - \log J = \frac{2}{3} \left\{ \left(\frac{aax + \beta by + \dots}{aa + \beta b + \dots} \right)^3 - \left(\frac{aax^3 + \beta by^3 + \dots}{aa + \beta b + \dots} \right) \right\} + \frac{2}{5} \left\{ \left(\frac{aax + \beta by + \dots}{aa + \beta b + \dots} \right)^5 - \frac{aax^5 + \beta by^5 + \dots}{aa + \beta b + \dots} \right\} + \text{etc.}$$

This difference must *always* be rather small in practical cases ; but, whether it be so or not, the correct method is that of aggregate expenditures (or costs). The above formulæ shew that the price-ratio method, used in the way indicated, must always give results that are substantially identical with the aggregate-expenditure method.

It might be thought that, in certain cases, the two methods could be combined. There could, of course, be no objection to so doing, provided the different results be properly weighted. To ascertain the proper weights to apply, however, is not a simple matter, and it is preferable to include all necessary items in the composite-unit, for what they are worth, and to employ the aggregate-expenditure method.

The great advantage of the aggregate-expenditure method over all others is, that one sees at every moment what is being done, and *if any item be uncertain, it is very simple matter to compute the effect of the uncertainty, and to see what its influence is on the result.* With price-ratio methods we are *working throughout "in the dark,"* and it can only be because of this that some economists have ventured the opinion that one may neglect weights altogether. This proposition arises from a wrong apprehension of the essence of the problem.

Price-indexes can be deduced accurately, and as has been shewn, have a definite significance when properly ascertained. Even if, for individual researches, price-ratios are ascertained (in order for example to follow the price-movement of any particular commodity) it is still desirable to find a price-index upon definite quantities of the individual items and applying the prices to these for the different dates compared, to base the price-indexes on the aggregate-cost.

15. Practical difficulties in obtaining accurate price-indexes.—The most pressing difficulty, as regards obtaining an accurate price-index, arises from that of obtaining accurate records of price. Were it not excluded by its magnitude, the ideal method would be to obtain for each commodity the actual quantities sold at each individual price, so as to get the "frequency-distribution" according to price, and thus to find from this the average price over all.* This, however, is quite impracticable, and it becomes therefore necessary to adopt a method that may be expected to give substantially, though not exactly, the same result. If the difference between the ideal and practicable methods is quite negligible, the theoretical defects of the latter are of no moment. By selecting in each locality (village, town, city, etc.) a sufficient number of establishments to cover the ordinary range of fluctuation of prices, and using the establishments patronised by the greatest number of inhabitants (rejecting, for example, those who sell only the finest class of goods as well as those who cater for the lowest class), we are able to obtain what is probably very approximately the weighted-mean of the prices : and this is doubtless a *very* close approximation to what would be found by the larger and impracticable operation. In the working sheets, one of which is necessary for each commodity, there is thus a continuous record, not only of the prices given by a single establishment, but by all establishments, and these records are available for comparisons one with another at each date or for successive dates. By this means any peculiarity can be at once detected, and if due to error is easily corrected on enquiry.

* In certain cases, however, we might need to base our deductions on the "mode" or predominating values not upon the average values.

In principle, the selected establishments automatically constitute a "fair sample" of the prices throughout the community immediately concerned. Strictly speaking, prices so found probably constitute the *mode* rather than the mean of the prices; in other words they represent the price paid by the greatest number rather than the price paid on the average. The difference, however, between the average and the most frequent or predominating price (the "mean" and the "mode") is probably quite negligible, even if it were without question desirable to adopt the mean; and moreover, whatever the difference, it is doubtless very nearly the same in amount and sign on each occasion. For this reason *errors or rather differences of this type have ordinarily no sensible effect on a price-index*. Suppose, for example, the true result was even as much as 10 per cent. in error by defect on one occasion, and 17 per cent. by defect on another, and that the item in which this occurred was about the order of 3 per cent. of the whole value of the composite-unit. A result of this kind would be given if, for example, the composite-unit consisted of 3 lbs. of the item under consideration, and if 10d. per lb. were entered instead of 11d. per lb. on the first occasion, and 18d. per lb. instead of 21d. per lb. on the second occasion. Thus if the aggregates for the two occasions were given as 1170 and 1920 they should have been 1173 and 1929. Thus the price-indexes would be:—

Erroneous = $1920 \div 1170 = 1641.0$; Correct $1929 \div 1173 = 1644.5$; a difference of only 3.5 in 1640, or say 2.1 per 1000.*

Returning to the question as to whether the use of the "average" or "mode" is preferable, it may be observed that in investigations of the character under review, we are concerned rather with the usage of the greatest number than with the usage of the average, so that those in circumstances of penury or luxury are advisedly excluded as of relatively minor sociological interest.

It may be observed that errors of price are readily detected: this is illustrated in the following example from actual returns from persons K, L, M, and persons P, Q, R. The following two series of returns were sent in by persons supplying prices on 15th of each month, the first for potatoes, the second for kerosene:—

Persons.	June.	July.	Aug.	Sept.	Persons	June.	July.	Aug.	Sept.
K	1/-	10½d.	10½d.	10½d.	P	2/4	2/4	2/4	2/4
L	1/-	10½d.	10½d.	1/6*	Q	2/3	2/3	2/3	2/3
M	1/-	1/-	1/-	1/-	R	2/4	2/4	7d.†	2/4

As each person furnishing a return is required to state if there is anything special in the price, the note to the price marked * above, happened to be that the price was a "cut-price:" hence if a difference was to be expected the price given should have been lower than the others. On referring the question back to the person concerned it turned out that, by an oversight the price was for 28 lbs., not for 14 lbs. Thus the correct price was 9d. In the second case, the price-marked†, the returns for August disclosed the fact that one retailer quoted 7d. per gallon for kerosene compared with 2/4 and 2/3 quoted by the other retailers. On inquiry it was discovered that by mistake the price *per quart* had been quoted instead of the price *per gallon*. These are typical examples of the way in which the continuous record on the "working" sheets enable accuracy to be secured.

16. **Character of items in composite-unit.**—The items in a composite-unit are of three principal kinds:—

- (i.) Those which are immediately *wholly* consumed in the act of using; such, for example, as foods, fuel, lighting, rent, amusements, educational expenses, etc. (Strictly those that are consumed rapidly).
- (ii.) Those which, immediately, are only partially consumed in the act of using, such as boots, hats, clothing generally, instruments of locomotion, carriages, motors, etc. (Strictly those that are consumed slowly.)
- (iii.) Those which are not consumed but which represent past expenditure that would otherwise bring interest, for example, houses, land, etc., life memberships, and entrance-fees giving right to benefits for life, etc.

* Or more generally 10 per cent., 17 per cent. and 3 per cent. = 0.10, 0.17 and 0.03. The effect would be $0.1 \times 0.03 = 0.0030$ and $0.17 \times 0.03 = 0.0051$: thus the two aggregates when corrected differ only 3.0 and 5.1 respectively in 1000. The purchasing-power deduced from these (ratio of the corrected numbers) differ only $1005.1/1003.0 = 1.0021$, or 2.1 per 1000.

The different characters of these several kinds of items which go to make up a composite-unit must be considered in determining the weight to be attributed to the items, but they must virtually all be reduced to the quantity used in some unit of time; for a unit, number of individuals, say per average person per week (or per month, per year, etc.). Thus, taking 1 "average person" and 1 month as the units, we need for (i.) the quantity of each food used, the cost for fuel, lighting, rent, amusements, education and so on, *per "average person"* per month.

For (ii.) we require the length of time the articles last with all circumstances concerning their maintenance in use. Let us suppose, for example, that a pair of boots costs 22/6, and with one repairing at 5/6 lasts 8 months: the cost would be $22\frac{1}{2} + 5\frac{1}{2} \div 8 = 3/6$ per month (neglecting interest charges, which at most would increase this expenditure to about 3/7 or 3/8). Similarly, if a collar cost 1/- and lasted afterwards for 23 launderings (that is 24 periods in all), and the laundering cost 2d. per occasion, the real cost for collars would be $(12d. \div 24) + 2d. = 2\frac{1}{2}d.$ for the average length of time the collar is worn. If this be one day, the cost for collars would be $7 \times 2\frac{1}{2}d. = 1/5\frac{1}{2}$ per week. If the collar cost 2/., the other facts being the same, the cost is $(24d. \div 24) + 2d. = 3d. \times 7 = 1/9$ per week: that is, although the difference in price was 100 per cent. on the cost at the first date, the real rise (for the composite-unit) would only be as 5 : 6, that is, 20 per cent.

Similarly, if a shirt lasted 19 launderings after the first, or say 20 periods, each laundering costing say 4d., its original cost being at one date say 4/., and at a later date 6/., and if it be worn two days between each laundering, the cost for the periods commencing at the two dates would be $(48d. \div 20) + 4d. = 6.4d.$ each laundering, and $(72d. \div 20) + 4d. = 7.6d.$ each laundering; that is in pence, 22.4 per week and 26.6 per week respectively. Thus, although the rise in *initial cost* was 50 per cent., the actual increase in the *usage cost* of shirts to the wearer is only as 6.4 is to 7.6, that is 16 : 19 or 18.75 per cent.

If facts of this character are *not* taken into account, *very erroneous results will arise in applying the rise in prices to questions of the cost-of-living*. The above illustrations shew that all circumstances tending to lengthen usage, and all costs of maintaining the commodity in a state of fitness for use are part of the consideration of its *real* as compared with its *nominal* cost. It is, therefore, important not to confound the two in price-indexes designed to be applied to questions of the cost of living.

In regard to (iii.), if an expenditure be incurred which confers some benefit virtually in perpetuity (e.g., land appropriately used, a farm, etc.), the ordinary rate of interest on its capital value may at least be allowed as its current or usage value. Thus suppose an expenditure of £1000 is incurred, the interest thereon, payable annually, being 5 per cent., the money-value of the benefit may be regarded as £50, which for a weekly-unit is equivalent to $£50 \times 7 \div 365\frac{1}{4} = 19s. 2d.$ per week. If this is to be regarded as accounted for either weekly or monthly (to accord with the period for the other elements of the composite-unit) it will be a somewhat smaller sum, for example, if it is considered to be provided monthly say about 18s. 9d., or if weekly 18s. 8½d. (the difference being due to a compound-interest correction).

A common case which would have to be accounted is where, say, a workman's house costs say £400; its maintenance costs £4 per annum each year thereafter. Let us suppose the house to be valueless (of negligible value), at the end of 25 years. If we could disregard interest questions over so long a period as in the illustration of (ii.) above, the total cost would be $£400 + 24 \times £4 = £496$. For this a benefit exists for 25 years = 1304 weeks; hence this would be equal to 7s. 7½d. per week.

Over so long a period, however, *the interest element cannot be ignored* as in (ii.), because it sensibly affects the question of the value equivalent to the outlay. If we assume an interest rate of 5 per cent. (payable annually), the whole outlay is equivalent to £400, together with an annuity of £4 payable at the end of every year for 24 years. The return equivalent to this is an annuity payable say weekly (this is virtually a continuous annuity), extending over 25 years. This gives a value of 12s. 1d. per week. If we should have, in addition, to take account also of the value of the land on which the house was situated, we should have to add, if this were held in fee-simple, the value of the interest only: and if this were to be accounted weekly it would be somewhat less than the annual value of the interest. For example, if the land were worth £200 (interest payable annually at 5 per cent.), it would mean 3/10 per week, or allowing for its being accounted weekly, say 3/9 per week.

In a looser kind of way, property owned can always be credited with what it would bring in the way of rental, less costs of maintenance: hence for practical purposes the rentals of properties of like kind could be assumed to apply.

17. **Changes in composite-unit.**—The composite-unit, on which a price-index has to be based, may become permanently inapplicable, in the course of time, through changes in ordinary usage; or it may become temporarily inapplicable in abnormal times, for example, when through famine, war, etc., a commodity is not available or is so high-priced that substitutes must be found. In both cases price-indexes in the ordinary sense become unmeaning. This is seen at once by considering an extreme case. We could not, for example, compare price-indexes for a community whose staple diet consisted of wheaten bread, meat, butter, etc., with one whose staple diet consisted of rice, fish, etc. We have, therefore, to realise that the significance (or the reality of the meaning) of price-indexes disappears in proportion as the regimens for two dates or for two communities, etc., materially differ. As already pointed out in such instances, we can have only a pseudo-continuity. This can be clearly represented by the following scheme:—

	Commodities disappearing.	Common to both dates.	Newly introduced.
Date 1800	A B C D E	F G H I J K L M N O P Q	
Date 1900		F G H I J K L M N O P Q	R S T U V W

From this schematic representation we can see at once that if the number of commodities disappearing (A to E), and the number of new commodities introduced (R to W) are small compared with the number of commodities common to the two dates (viz., F to Q), the price-index is still fairly significant.

It has to be remembered that as commodities disappear there is no sense in which price can be attributed to them, and that, while they are disappearing, price may become uncertain. There are commodities for example which have a vogue at a particular time. Human fashions change and these commodities disappear. Initially their prices are fanciful; intermediately they may be said to be normal, and in the disappearing stage they are irregular and uncertain. Hence there is no way in which we can make price-indexes, extending over long periods of time, significant in relation to one another over such periods.

It is, of course, obvious that an unequivocal price-index could be determined on the basis of the commodities F to Q in the above example, and this would have some value for remotely distant periods (e.g., 1800 and 1900), but for current usage a price-index based upon the limited number when a larger number is *ex hypothesi*, available, would be subject to the criticism that it did not represent the actual usage of the community. A method, therefore, must be devised for passing from one regimen to another, and we shall next consider that problem.

The change may moreover be not merely one in the kind of commodities but also a change in proportionate usage of the original commodities, owing to the introduction of substitutes, as for example, the replacing of, say, oatmeal by some form of wheaten or maize-meal as a breakfast-food. It has to be remembered that, statistically, the fact of commodities changing is known necessarily after the change has occurred; for that reason it is possible to effect revisions of price-indexes in such a manner that they may become of higher value in respect of quasi-continuous record.

18. **Revision of price-indexes to secure high accuracy over long periods.**—Continuity of price-indexes of the highest order of precision can be secured by varying the regimen so that at all times it represents actual usage.

Whatever the actual usage may be at two dates O and N, say, the *minimum change* of that usage would be when each element is made to increase (or diminish) by equal amounts in each interval of time (year, say). This is expressed by the quantities for the successive N years being made for each item as follows:—

Date	0	1	2	3		N
Quantity of item	a	$a \pm x$	$a \pm 2x$	$a \pm 3x$		$a \pm nx = a''$

Thus the amount to be added each year is $(a' - a)/N$, and this includes the cases when either $a' = 0$ or $a = 0$, that is when either a commodity disappears from usage or a new one is introduced. Thus, for the index of date 1, we may use the series of quantities changed $(1/N)$ th part (or we may use the original series; the former is usually preferable because when a regimen or composite-unit has been ascertained

it is referable to a past period). By this process the pseudo-continuity of the whole series of price-indexes is made of the highest possible value since the basis is changed from year to year by insensible steps; but it requires always that the prices should be available. In order to shew the effect of such a method we may take what may be regarded as an extreme case; that is, one which will accentuate the differences between the ordinary method and the method to be used for revisions, the purpose of which is to make the pseudo-continuity as satisfactory as is possible.

In the table hereunder let the commodities in the composite-unit be those shewn in column (I), the denomination of the unit being as shewn in column (II.). Seven commodities, not used at the first date (1908), (*i.e.*, are not in composite-unit A), are used at the last date (1913), and seven which existed originally (*i.e.*, are in A) disappear from the composite-unit of the last date (1913). (*i.e.*, from F). We make the changes for the intermediate years linearly but by whole integers; that is, as near as this limitation will admit, the change from regimen A to regimen F is by equal amounts from year to year. We have also arranged that the items in F (*i.e.*, for 1913) shall have the same aggregate cost at 1913 prices as the items in A at the 1913 prices. This is to preserve the continuity, as previously indicated.

Table Shewing Marked Changed in the Composite-unit.

Commodity. (I.)	Unit. (II.)	A. 1908.	B. 1909.	C. 1910.	D. 1911.	E. 1912.	F. 1913.
Bread	lb.	..	167	334	501	668	836
Flour	"	300	242	184	126	68	10
Tea	"	..	6	12	18	24	30
Coffee	"	30	24	18	12	6	..
Sugar	"	300	320	340	360	380	400
Rice	"	50	40	30	20	10	..
Sago	"	8	6	5	3	2	..
Jam	"	..	15	29	44	59	73
Oatmeal	"	35	28	21	14	7	..
Raisins	"	..	3	6	9	12	14
Currants	"	14	12	9	6	3	..
Candles	"	30	24	18	12	6	..
Soap	"	50	53	56	59	62	64
Potatoes	"	1,000	897	794	691	589	487
Milk	qrt.	400	360	320	280	240	200
Butter	lb.	..	11	22	33	44	55
Cheese	"	..	3	6	9	12	15
Eggs	doz.	9	11	13	15	16	18
Bacon	lb.	20	24	28	32	36	40
Beef	"	..	70	140	210	280	350
Mutton	"	750	600	450	300	150	..
Rent	week	46	46	46	46	46	46

We shall first shew the effect of adopting the different regimens or composite-units, *viz.*, A to F, as bases. By applying actual average Australian prices for the commodities to each of these and summing them we obtain for the total costs the following results, *viz.* :—

Actual Cost of each Composite-unit at Average Current Prices in Australia during the Years 1908-1913.

Composite Unit.	1908.	1909.	1910.	1911.	1912.	1913.
A	148339	144607	153940	158472	171482	171029
B	148250	144787	153684	158203	171019	171130
C	148080	144888	153347	157855	170474	171456
D	147924	145002	153025	157518	169940	171190
E	147641	144985	152562	157047	169273	171109
F	147374	144988	152125	156597	168627	(171027)

From these actual aggregate costs during the years 1908-1913 for the series A to F of groups of commodities, we can analyse the effect of the different bases upon the price-indexes. Let us first observe the difference of the effect of using one or the other of these various composite-units throughout the whole period. Owing to the fact that the variations of price are not uniform throughout this will be best shewn by making say A = say 10000, and then seeing what ratio the expenditures on (or cost of) the others bear to this. We do this for each year. The results are as shewn in the table hereunder.

Relative Money-value of each Composite-unit in each Year, 1908-1913.

Composite Unit.	1908.	1909.	1910.	1911.	1912.	1913.	Average
A	10000	10000	10000	10000	10000	10000	10000
B	9994	10012	9983	9983	9973	10006	9991
C	9983	10019	9961	9961	9941	10025	9981
D	9972	10027	9941	9940	9910	10009	9966
E	9953	10026	9910	9910	9871	10005	9945
F	9935	10026	9882	9882	9834	10000	9926
Averages ..	9973	10016	9946	9946	9921	10007	9968.2

The most striking fact is that the different regimens (composite-units A to F) give results which are so nearly equal in value for the same year. Yet these units are, as we have seen, by no means identical. It is to be remarked that they vary in different ways in the several years. Thus for 1909, A is cheapest, but dearest for any other year except for the year 1913, when it equals F, but is dearer than B, C, D and E. This equality with F, however, as already pointed out, was secured by so taking the number of units of the new regimen that it would give that result in the final year, and this has the effect of keeping them nearly equal in cost throughout. This was done in order to secure aggregates that preserved the pseudo-continuity in the index-numbers. It may again be repeated that *the actual number of units do not affect the index-numbers* : it is only the proportions subsisting among them which can produce a variation in the determination of a price-index.

Next let us see the effect of applying the several regimens A to F throughout the period 1908 to 1913. This is best effected by making the result for 1908 equal to 10000 throughout. Thus we get :—

Price-indexes with Different Composite-units as bases. (1908=10000).

Composite Unit.	1908.	1909.	1910.	1911.	1912.	1913.	Aver- ages.
A	10000	9748	10378	10683	11560	11530	10649
B	10000	9766	10367	10671	11536	11543	10647
C	10000	9784	10356	10660	11512	11579	10648
D	10000	9802	10345	10649	11488	11573	10643
E	10000	9820	10333	10637	11465	11590	10641
F	10000	9838	10322	10626	11442	11605	10639
Averages	10000	9793	10350	10654	11500	11570	10644.5

The striking fact is that the indexes calculated by means of any one of the different regimens are almost identical, thus shewing that *considerable variations in the composite-unit do not produce large changes in the price-indexes*. This important fact is generally overlooked, and is often ignored by investigators.

19. **Price-indexes with a periodic change of the composite-unit.**—If it be assumed that, on the average, a certain composite-unit applies to two periods of time in succession, it is obvious from what has preceded that it will be the best basis on

which to estimate the price-index ratio. Theoretically, the smaller the periods the better will be the result. It is also evident from what has preceded that no sensible advantage would be secured by making these periods less than 1 year. We shall therefore, from the preceding table of expenditures, trace the consequence of making the change annually. This is done by dividing the numbers in heavy type in the table shewing the "actual cost of each composite-unit during the years 1908-1913," into the quantities next on the right. The quotients are the ratios of the price-indexes. Hence if we make the first 1000 and multiply by those successive ratios we get the various price-indexes on the basis that 1908 is 1000 (or any other unit or value we may choose to adopt). Thus:—

$$144607 \div 148339 = .974842; 153684 \div 144787 = 1.061449;$$

$$157855 \div 153347 = 1.029397; \text{ etc.}$$

Hence $.974842 \times 1000 = 974.842$; this by $1.061449 = 1034.745$; and this again by $1.029397 = 1065.163$.

In this way we get the results shewn on line 2, or variable A' in the table below. If we were to start with 148250 and use (in the same way throughout) the quantities under the lines, we should get the results shewn on line 3, or variable B'.

Again if we were to use the composite-unit A throughout, we should get the results on line 1, or if the composite-unit F, the results on line 4.

In regard to the price-indexes in the preceding table, it is self-evident that 9838 on line F is a less accurate result than any one of the three above it because the composite-unit, used in calculating it, represented actual usage not earlier than 1913, and for the same reason the values 11560 for 1912 or 11530 for 1913 are *less reliable* than any values below them, the composite units of actual usage in the years 1912 to 1913 differing greatly from A.

Various Estimations of Price-indexes, 1908-1913. (1908 = 10000).

Line.	Composite-unit	1908.	1909.	1910.	1911.	1912.	1913.
1	A	10000	9748	10378	10683	11560	11530
2	Variable A'	10000	9748	10347	10652	11492	11616
3	Variable B'	10000	9766	10348	10640	11468	11632
4	F	10000	9838	10322	10626	11442	11605
5	$\frac{1}{2} (A + F)$	10000	9793	10350	10654	11501	11565

From the considerations above indicated, it is readily seen that the results shewn on either line 2 or line 3 should be adopted in preference to those on lines 1 or 4, but whether those on line 2 should be preferred to those on line 3 or not, will depend on which regimen should be regarded as the most nearly that covering the two periods compared.

Finally, if we were to apply the mean of the two regimens A and F, the expenditures would be 147960, 144896, 153135, 157635, 170163, and 171120, and these would give the results on line 5. These are obviously *accurate enough to be adopted* for all practical purposes, and shew that we are not really concerned with small variations in a regimen, and that *meticulous accuracy in regard thereto is quite unnecessary*. The mean would correspond to the mean of the items on vertical lines C (1910) and D (1911) in the table "shewing marked changes in the composite-unit." In brief, so long as the composite-unit represents the mean usage over any limited period (such as 5 years or 10 years), *the one unit may be applied throughout* and may be abandoned for a new unit for the period next following on. This is self-evident if it be remembered that the preceding tables might be taken to represent changes which take place every quinquennium or decade.*

* The general case has been fully established by my article in Labour and Industrial Report, No. 1, Commonwealth of Australia. Appendix, sections 8, 10 and 11, pp. xlix. to lv., Dec. 1912.

20. **Omission of items from composite-unit.**—The omission of items from a composite-unit is necessary for the following reasons, viz., that :—

- (i.) They cannot be specified and identified with sufficient precision for compared places or periods.
- (ii.) The quantity used is so insignificant that their inclusion or omission does not make a sensible difference in the results.
- (iii.) They are uniformly constant in value.

The effect of ignoring a number of commodities is *virtually* equivalent to assuming that their variations of price are, *on the average*, identical with the variations, on the average, of those included.

In regard to (i.), it is to be observed that if the limits of uncertainty are great, it is preferable to omit the item. This has already been partially considered in section 1 of this chapter. The principle of gauging whether the results should be used or not is the following :—

- (a) the range of uncertainty must be specifiable both in the positive direction and in the negative.
- (b) If the price-index, deduced by omitting the item, lies outside the range of the price-indexes when it is included, the omission is unsatisfactory, and the mean of the range should be adopted, with the uncertain item (or items), introduced.

In regard to (ii.), it may be said that the omission is desirable since the introduction of a number of insignificant items greatly increases the volume of work with no sensible advantage.

In regard to (iii.), whether constant items should be introduced or not, this depends upon whether they may be set off in a general and somewhat loose estimate—any other being impossible—against items which have probably departed from the price at the earlier date *beyond* the mean amount. If not, the following scheme of modifying the price-index deduced may be adopted :—

Let E denote the total *value* of the composite-unit, the quantities of the items being now, however, not merely relatively, but as near as possible absolutely correct : let K denote the aggregate of constant expenditure and s the portion which may be supposed to vary as E . Then, putting $s = S_0 / E_0 = S_1 / E_1$, wherefore $S_1 = S_0 E_1 / E_0$, the proper index is given by—

$$(15) \dots I = \frac{E_1 + S_1 + K}{E_0 + S_0 + K} = \frac{E_1 (1 + s) + K}{E_0 (1 + s) + K}$$

which is readily calculated, and does not require that S and K should be very exactly known. These could of course be expressed as ratios. Suppose, for example, E_0 is about 60 per cent., S_0 about 30 per cent., and K about 10 per cent.* at the original date, this last expenditure being *ex hypothesi* unchanging. In the ten years 1907-1917 the ratio of E_1 to E_0 became 1.5, that is, the price-index increased 50 per cent. Thus if E_0 be 100, $E_1 = 150$, and the index as ordinarily calculated is 1500, that is, 1000×1.5 . The adjusted index, *on the above assumption*, would be (supposing the unrecognised portion of the total (S) to increase in the same ratio as that for which the record exists, but excluding, however, the constant 10 per cent.) :—

$$I' = \frac{150 + 75 + 10}{100 + 50 + 10} \times 1000 = \frac{235}{160} \times 1000 = 1468.8 \text{ instead of } 1500.$$

Suppose that the unrecognised portion S were *also constant*, the result would be—

$$I'' = \frac{150 + 50 + 10}{100 + 50 + 10} \times 1000 = \frac{210}{160} \times 1000 = 1312.5 \text{ instead of } 1500.$$

Further, suppose that the unrecognised portion actually increased 100 per cent., i.e., became as much as twice what it originally was, the result would be

$$I''' = \frac{150 + 100 + 10}{100 + 50 + 10} \times 1000 = \frac{260}{160} \times 1000 = 1625.0 \text{ instead of } 1500.$$

The first result (I') is, of course, the mean of the two last ; that is, it is $\frac{1}{2} (I'' + I''')$.

* This is about its maximum value.

Where indexes are given only to three places (as is often the case) the first difference is clearly seen to be small: thus we have 150 and 147 in the example first considered: the constant element being 10 per cent. of the total expenditure at date 0, the reduction of the price-index has been only about 2 per cent.

In the latter cases which may be regarded as extreme possibilities, the effect has been to reduce the price-index 150 to 131, or to increase it to 163; that is, to reduce it 13 per cent. or increase it 9 per cent. This, however, is the consequence of two things, viz., to constancy in the 10 per cent., together with a change in the 30 per cent. (at date 0) proportionally different to the change in the 60 per cent. (at date 0).

21. Variations of price-levels.—One form of *price-index*, to which reference has been made in section 5, and which serves a useful purpose, may be formed in the following way:—

Let the quantities of exports (or imports) of a given period have attributed to them (a) the actual prices obtaining for the period under review, and (b) the prices they had on a previous period with which it is desired to compare them. (More strictly it is a comparison of the present with the past.) This comparison informs us what *would have been* the aggregate value of the commodities of the former period if, their prices being as they actually were, they had been in volume what they are in the second period, and therefore the ratio of this to the total for the second period. To distinguish these from price-indexes generally, we shall call them *price-levels*.* The basis of comparison is thus always made the "regimen" or composite-unit of the period which is to be compared with some former period. It is obvious that the indexes thus obtained are not comparable among themselves, because the composite-unit used each year is special to itself; that is, for three successive years, it would be:

a of A + b of B + c of C + etc.; a' of A + b' of B + etc.; a'' of A + b'' of B + etc.

The small letters denoting quantities and the capital letters commodities.

Obviously we can make any one of the sets of quantities a, b, c, etc.; a', b'', c', etc.; the basis. If we postponed the comparison till the end of a quinquennium decade, etc., the *best* basis would be the mean (geometric) of the whole period, or we would "step up" the composite-unit as described in section 18.

Example of price-levels.—Price-levels of exports for Australia are prepared for five groups of commodities, viz., those embraced under the following headings:—

- (i.) **Agricultural production.**—19 items embracing—1, fodder; 2, fruits repulped; 3, unprepared grain, such as—4, barley; 5, beans and peas; 6, maize; 7, oats; and 8, wheat. Prepared grain such as—9, bran, pollard and sharps; 10, flour; and 11, oatmeal; 12, hay and chaff; 13, hops; 14, jams and jellies; 15, linseed cake and oil cake; 16, onions; 17, potatoes; 18, cane sugar; 19, wines (fermented); 20, wines, sparkling.
- (ii.) **Pastoral productions.**—16 items embracing—21, lard and refined fats; meats—22, bacon and hams; preserved by cold process—23, beef; 24, mutton and lamb; 25, pork; 26, rabbits and hares; 27, meat preserved in tins; 28, miscellaneous meats; 29, hair; 30, gluepieces and sinews; skins including—31, cattle and horse; 32, rabbit and hare; 33, sheep skins with wool; 34, tallow; wool, viz., 35, greasy, and 36, scoured and tops.
- (iii.) **Dairy productions.**—6 items, embracing—37, butter; 38, cheese; 39, eggs; 40, honey; 41, preserved milk; 42, beeswax.
- (iv.) **Mineral productions.**—9 items, embracing—43, britannia metal, etc.; 44, coal; 45, coke; 46, copper ingots and matte; lead including—47, pig in matte; and 48, sheet and piping; 49, salt; 50, kerosene shale; 51, tin ingots.
- (v.) **Miscellaneous.**—19 items, embracing—52, ale and beer; 53, tanning bark; 54, biscuits; 55, candles; 56, Portland cement; 57, confectionery; 58, copra; 59, unrefined glycerine; 60, gums; 61, lime-juice; 62, manures; oils, etc.; in bulk including—63, coconut; 64, linseed; 65, tallow; 66, sandalwood: soaps, 67, ordinary; and 68, perfumed; 69, pearlshell; and 70, tortoiseshell.

* This method is that adopted by the British Board of Trade, and hitherto in the Official Year Book of the Commonwealth of Australia. Its defects are pointed out later.

These groups represented about 84½ per cent. of the total export of merchandise during 1915-16, though the actual items were only 69 out of 545; the balance of 476 items represented only the small value of about 15½ per cent.

The ratio of the values of the groups to the value of the total varies of course with the prices. This is shewn in the following table :—

Price-levels of 1915-16 compared with 1901, on Actual Exports of 1915-16.

Class of Production.	No. of Items.	VALUE OF EXPORTS OF 1915-16.				Price-levels, 1901 = 1000.
		At 1901 Prices.		At 1915-16 Prices.		
		£	%	£	%	%
Agricultural ..	19	5,478,627	13.68	10,567,031	16.42	1928.8
Pastoral ..	16	21,355,362	53.31	33,570,881	52.14	1572.0
Dairy ..	6	776,926	1.94	1,159,857	1.80	1492.9
Mineral ..	9	5,116,696	12.77	7,894,448	12.26	1542.9
Miscellaneous ..	19	1,108,267	2.77	1,195,461	1.85	1078.7
Total Nos. used	69	33,835,878	84.47	54,387,678	84.47	1607.4
Remaining Nos.	476	6,221,006	15.53	9,999,624	15.53	1607.4
Total exports ..	545	40,056,884	100.00	64,387,302	100.00	1607.4

Since the quantities are the same (those of trade-year 1915-16) in both cases, the columns of percentages shew that the mere variations of price as between 1901 and 1915-16 make sensible, though not large difference, in the ratio of the value of each class of production to the total value.

The large differences in the price-levels (final column) shew that, while there was a general increase in prices for the different classes of production, it was by no means similar in amount.

Price-level comparisons could, of course, also be made on the basis of the actual exports of the earlier period, attributing thereto the prices of the later period. This will, of course, give a different result, and in order to shew the nature of the difference, the following table has been prepared :—

Class of Production.	QUANTITIES AS IN TRADE-YEAR 1901.			QUANTITIES AS IN TRADE-YEAR 1915-16.		
	Values at 1901 Prices (as Recorded).	Values as at 1915-16 Prices.	Price-levels 1901 = 1000.0	Values at 1901 Prices.	Values at 1915-16 Prices (as Recorded).	Price-levels 1901 = 1000.0
	£	£		£	£	
Agricultural	4,508,717	8,315,780	1844.4	5,478,627	10,567,031	1928.8
Pastoral ..	18,945,409	30,321,569	1600.5	21,355,362	33,570,881	1572.0
Dairy ..	1,486,033	2,213,902	1489.9	776,926	1,159,857	1492.9
Mineral ..	3,161,806	3,979,150	1258.5	5,116,696	7,894,448	1543.1
Miscellaneous	667,721	747,121	1118.9	1,108,267	1,195,461	1078.9
Total ..	28,769,686	45,577,522	1584.2	33,835,878	54,387,678	1607.4

Price-levels such as these give, of course, an *unequivocal* answer only to such a question as "what would have been the value of the imports of any particular period if the quantities have been what they were at another, e.g., a later period,"

or *vice-versa*. But if they are to be applied in any endeavour to compare the quantities of export what may be called the "generalised quantity" of the exports, they do not give an unequivocal answer, and hence are not of the same value as price-indexes. Thus:—

$$\frac{\text{Value of exports 1901 at 1901 prices}}{\text{Value of exports 1915-16 at 1901 prices}} = \frac{£28,769,686}{£33,835,878} = \frac{1000.0}{1176.1};$$

which implies that the generalised quantity was 1.1761 greater in 1915-16 than it was in 1901, while on the other hand the ratio—

$$\frac{\text{Value of exports 1901 at 1915-16 prices}}{\text{Value of exports 1915-16 at 1915-16 prices}} = \frac{£45,577,522}{£54,387,678} = \frac{1000.0}{1193.3};$$

implies that the generalised quantity of exports was 1.1933 greater in 1915-16 than in 1901. It is obvious that there is no reason for preferring one of these results to the other, and the difference between the two is not insensible.

Price-levels, therefore, cannot be safely used if it be desired to compare with any precision the 'generalised quantities' of the exports for two different years. They furnish a rough idea, of course, and that is all.

Nor can we take the mean of these two results as satisfactory, viz., 1000.0 : 1184.7, as will appear later.

22. **Advantages of a price-index over a price-level.**—Consistently with what has preceded, we shall call a price-index (in contradistinction to a price-level) a result furnished by adopting, for the quantities to which the prices of two different dates are to be applied, a common basis, which ordinarily should be as near the actual quantities on the one date as on the other. Two methods are suggested; one is to adopt the average quantities taken over the period in question, the other to take the average quantities only for the two years to be compared. We shall adopt a later method and shall put in comparison therewith a variation of the former type, viz., the averages for two periods, viz., 1906-10 inclusive and 1911 to 1915-16 inclusive.

Comparison of Price-indexes for Exports on Several Bases.

CLASS OF PRODUCTION.	QUANTITIES ADOPTED AS BASIS (COMPOSITE-UNIT) ARE THE ANNUAL AVERAGES OF—								
	Years 1901 and 1915-16.			Yrs. 1906, 1907, 1908, 1909, 1910.			Yrs. 1911, 1912, 1913, 1914, 1915-16.		
	At 1901 Prices.	At 1915-16 Prices.	Price-index.	At 1901 Prices.	At 1915-16 Prices.	Price-index.	At 1901 Prices.	At 1915-16 Prices.	Price-index.
Agricultural	4,990,417	9,442,904	1892	2,752,047	5,108,245	1856	2,226,963	4,062,260	1824
Pastoral	20,350,306	32,296,652	1587	24,645,201	39,330,601	1596	27,533,825	43,634,000	1585
Dairy ..	2,235,099	3,317,734	1484	2,845,746	4,218,527	1482	2,726,528	4,074,991	1494
Mineral	4,142,930	5,942,739	1434*	5,711,553	8,010,463	1403*	5,466,074	7,992,513	1462*
Miscellaneous	888,353	971,071	1093	888,523	992,317	1117	961,886	1,042,545	1084
Total	32,607,105	51,971,100	1593.8	36,843,070	57,660,153	1565.0	38,915,276	60,806,309	1562.5

* This difference was due mainly to the difference in the part that lead played in the exports.

The above table shews that the price-index, based upon the quantities which were the mean of those of the years compared, was 1593.8: that based upon the mean, of the quantities for the years 1906 to 1910, was 1565.0; and that based upon the mean of the years 1911 to 1915-16 was 1562.5. The mean of the two latter, viz., 1563.7, differs very little from either, but differs sensibly from the first, viz., 1593.8.

The price-indexes based upon the quantity-averages over the whole of the years 1906 to 1915-16, are the means of the two latter price-indexes in the table above; that is they are 1840, 1590, 1488, 1432, 1100, and for the total 1563.7. We thus see that, when the quantities are averages taken over a sufficient period of time, the results are almost identical, and, therefore, we can ascertain, in an unequivocal way, the ratios of the "general quantity" of the exports on any two occasions.

This satisfactory agreement is dependent upon the fact that, when calculated on the basis of the quinquennial averages of quantities, the bases themselves are likely to be nearly in agreement. There appears to be a rough periodicity in exports, the

period being about $7\frac{1}{2}$ years, and consequently not only are the sharp differences characterising individual years smoothed out, but so also is the systematic fluctuation—at least in part. Thus, the quantity-groups of the 1906-10 average (giving one basis) do not differ greatly from the quantity groups of 1911-15-16 average (giving the other basis). They were not, however, identical as may be seen by comparing one basis with another. This difference is reflected in the ratios of the prices of one group for the two years, to be compared according to one or the other set of quantities.

Thus the prices for 1901 agricultural production, on the 1906-10 quantities basis, and on the 1911-15-16 basis were respectively £2,752,047 and £2,226,963, their ratio being 0.8092. The prices for the same group for 1915-16 on the same two bases were respectively £5,108,245 and £4,062,260, their ratio being 0.7952, instead of 0.8092. The difference of either from the mean (0.8022), is small; it arises from the lack of absolute identity in the *mutual proportions* of the nineteen (19) individual items (see para. (i.) page 43) constituting the group. The comparison for the whole series is as follows:—

Groups.	Agri-cultural.	Pastoral.	Dairy.	Mineral.	Mis-cellaneous	Total.
Basis 1906 to 1910	.8092	1.1172	.9581	.9570	1.0826	1.0562
Basis 1911 to 1915-16	.7952	1.1094	.9660	.9976	1.0506	1.0546

The preceding figures shew that a price-index has a perfectly general and unequivocal significance, whereas a price-level is a very arbitrary form of comparison, and its use in any attempt to deduce the "generalised-quantities" of imports or exports is invalid. It will often, of course, give a rough approximation, whereas a price-index, based upon the average amounts of the commodities taken over an extended period, will give the most satisfactory result which it is possible to obtain. In so far as it is practicable to give any general form of expression to the quantity of exports, when values are available, the correction by means of a price-index is justified, and we are not justified in using a price-level as the means of correction.

We do not propose to consider here the nature of a hypothetical generalised quantity in detail. It will suffice to say that although it is intrinsically impossible to compare commodity-aggregates that are not identical in character, we can make a sort of pseudo-comparison on the supposition that both are of the nature of the mean composite-unit used in determining the price-indexes. It is in this sense that we ascertain the "relative-quantities." The validity of the comparison is based upon the fact that the commodities on each occasion do not differ materially from the mean; thus the comparison has considerable value, and is of the nature of an index. In any case it may be said that it is the only comparison possible.

23. **On the discontinuity of price-indexes.**—The fact that, with change of usage, price-indexes cannot—in any accurate sense of the term—be regarded as strictly continuous, has already been referred to. It has also been shewn that the nature of such continuity as is possible is that price-indexes, based upon aggregate costs or expenditure of composite-units are really fully comparable only when the composite-units are sensibly identical in character. Abnormal times, therefore, involving departures from ordinary usage constitute a similar difficulty. Consequently if, in order to represent actual usage, the composite-unit has to be materially changed, it may become necessary to indicate that the price-indexes during the abnormal period are not strictly comparable in the full sense of the term with the earlier and later price-indexes. We have shewn that the only valid basis of comparison between two years (say) in which the usage is different, is the mean-regimen of the two. Let us suppose that each year these regimens are changed, there will then be only one computation of aggregate expenditure for the first, and for the final year (i.e., when normal conditions, say, are restored), and two for each of the intermediate years. These aggregate expenditures may be denoted by

$$A_0; A_1, A'_1; A_2, A'_2; \dots; A_{n-1}, A'_{n-1}; A_n.$$

Thus the price-indexes* are given by—

$$(16) \dots\dots I_{01} = A_1/A_0; \quad I_{12} = A_2/A_1; \dots\dots I_{n-1,n} = A_n/A_{n-1}$$

Writing this *in intenso* we have—

$$(17) \dots\dots I_{0n} = \frac{A_1}{A_0} \cdot \frac{A_2}{A_1} \cdot \frac{A_3}{A_2} \dots\dots \frac{A_n}{A_{n-1}} = k \frac{A_n}{A_0}, \text{ say,}$$

It is obvious from this, that perfect continuity is *not* necessarily restored, when the regimen again becomes normal, but in all practical cases it must be nearly restored: i.e., k must be very nearly unity. For if we make the cost of $A'_1 = A_1$, $A'_2 = A_2$, etc., by changing all the quantities of the items in the same proportion, all the intermediate values in the above equation cancel. If they have not been made equal, suppose $A'_1 = mA_1$; $A'_2 = nA_2$, etc., then we have the above equal to—

$$(18) \dots\dots I_{0n} = \frac{A_1}{A_0} \cdot \frac{A_2}{mA_1} \cdot \frac{A_3}{nA_2} \dots\dots \frac{A_n}{sA_{n-1}} = \frac{A_n}{A_0 (m \cdot n \cdot \cdot s)}$$

s denoting the last factor necessary to make the denominator equal to the preceding numerator.

Hence $k = 1/(m \cdot n \cdot \cdot s)$, and ordinarily this must be nearly unity. We shall later illustrate this by an example. If k be not unity, it is obviously desirable to alter the intermediate values linearly so that the value of A_n/A_0 is the price-index I_{0n} . Then we have the highest degree of consistency attainable, and the price-indexes may be regarded as continuous.

This procedure may be called "closing up on to the normal values." In order to illustrate this process, and to shew that the results are *essentially* discontinuous, and further that the whole process of closing up is arbitrary, though the best possible, let us take an extreme case, viz., that illustrated in the following tables:—

Com- mod- ity.	Actual Usage, say, for 1 Week, in lbs.					Usage-basis for Calculating Price- indexes.					
	* 1914.	1915.	1916.	1917.	* 1918.	1913-4	1914-5	1915-6	1916-7	1917-8	1918-9
Bread ..	5	5	0	4	5	5	5	2½	2	4½	5
Meat ..	8	3	0	2	8	8	5½	1½	1	5	8
Rice ..	0	2	5	2	0	0	1	3½	3½	1	0
Fish ..	0	5	10	7	0	0	2½	7½	8½	3½	0

* 1913, 1919, 1920, etc., supposed identical with this.

The actual usage for the successive years is that given on the left-hand portion of the table. For the purpose of comparison we must take—as indicated hereinbefore—the means of the regimens of the adjoining years as the basis on which the price-indexes of one as compared with the other can be established. These are shewn in the right-hand portion of the table. Let us suppose that the prices for these years were as in the left-hand side of the table hereunder, then the aggregate expenditure would be as shewn in the right-hand portion of the same table.

* The notation I_{ai} denotes the price-index for year i with reference to year a as the basic year.

Prices and Aggregates of Expenditure.

Com- mod- ities.	AVERAGE PRICES (SAY PENCE PER LB. DURING YEAR :—							AGGREGATE-EXPENDITURE FOR EACH YEAR :—								
	1913	1914	1915	1916	1917	1918	1919	1913	1914.	1915.	1916.	1917.	1918.	1919.		
Bread	3.4	3.5	4.5	3.9	3.8	3.5	3.4	5860	6310 5535	7395 4860	.. 5285 4890	 5165	 7370	 5425	 6310	=1000 : 1076.8 =1000 : 1336.0 =1000 : 1087.4 =1000 : 1056.2
Meat	5.2	5.7	7.9	9.0	8.7	5.7	5.2	..	..	..	..	..	..	..	..	
Rice	..	2.5	3.0	3.1	3.3	3.0	..	1000	736.1	..	..	7370	5425	..	..	
Fish	..	1.6	2.0	2.5	2.8	2.0	..	..	..	..	..	..	..	5860	..	

For simplicity we make the prices in 1919 as in 1913, and hence if the price-index for 1913 is 1000 we should obtain 1000 also for 1919. The ratios in the above table are shewn on line "Factor (1)" line (ii.) in the table below. They give—by forming the continuous products—the results on line (iii.). If these are linearly changed by altering each by the multiples of $(1129.6 - 1000) \div 6$, that is multiples of 21.6, we get the corrected indexes on line (iv.).

These are equivalent to changing the factors (1) on line (ii.) into the factors (2) shewn on line (v.), and a comparison of the two shews the real amount of change required, which is by no means negligible in this extreme case, but would be ordinarily very small.

Correction of Price-indexes on Closing up on a Final Value.

(i.) Year	1913.	1914.	1915.	1916.	1917.	1918.	1919.
(ii.) Factors (1)*	1076.8	1336.0	1087.4	1056.2	736.1	928.7	
(iii.) Price-indexes as deduced	1000	1076.8	1438.6	1564.4	1652.4	1216.3	1129.6
(iv.) Price-indexes corrected	1000	1055.2	1395.4	1499.6	1566.0	1108.3	1000
(v.) Factors (2)*	1055.2	1322.4	1074.7	1044.3	707.7	902.3	
(vi.) Ratio of Expenditure	1000	1076.8	1261.9	901.9	881.4	925.8	1000

* For comparison Factors (2) do not agree with Factors (1) because of the necessity of the corrections.

That the ratios of the mere aggregates of expenditure *with changed regimens* are valueless, is shewn by line (vi.), which gives the relative expenditures on the mean regimens, the initial one being taken as 1000.

24. **Substitution of equivalent Items in a composite-unit.**—Whenever, in respect of purpose and quantity, any new item of a composite-unit is identical with that of the commodity which it wholly replaces, the price of the new commodity, may, for some purposes merely replace that of the old one, and in such a case the original mass-units would continue in force. Of course for other purposes this procedure would be invalid. It might, for example, be valid for index-prices which had reference to expenditure on the *cost of living*; but if the *standard-of-living*, either from the economic standpoint or that of food-value, were involved, this might or might not be invalid according to circumstances. We reach, therefore, the idea of a composite-unit that may be appropriately changed periodically without losing materially its significance.

In attempting, in any comprehensive way, to deduce price-indexes for commodities generally, or for some particular purpose, therefore, it may be necessary to arrange for the substitution—to some extent—of what may be called equivalent items, viz., items which, though not absolutely identical, may be regarded as identical

without in any way vitiating the deduced indexes. In other words we must consider the substitution of composite-unit 2 for composite-unit 1, which may be represented as follows :—

Elements of original unit (1) :—A B C D E F G H I J, etc.

Equivalent unit(2) :—A B C' D- E F' F" G H' H" H'' I J, etc.

in which the substitutions are C' for C ; F' and F" for F ; H', H" and H'' for H ; and so on ; the remaining elements—A B D E G I J, etc., being unchanged. The significance of such changes may be illustrated in the following way :—Suppose the usage of a community changed by abandoning C (or wheaten bread) for C' (or whole-meal bread) without changing the quantity used, C' could probably replace C, that is to say, the price of C' could be entered as if it belonged to the original element C of the composite-unit. For example, if in any community butter was completely abandoned for margarine, the margarine might be treated exactly as though it were butter, and its price entered. If the substitute *wholly* replaced the original commodity, and was used in like quantity, the mass-units (or quantities) of the items in the composite unit would remain unchanged, in which case the general economic effect might, *for most purposes, be regarded merely as a change in the price of the original items.* Of course if the quantity *used* be changed, the mass-unit of the commodity will be changed, and consequently for the date of change, two aggregate costs will have to be made out, one with the original item, and one with the substituted item (if, of course, their prices are not identical) the one for carrying the index up to the date, the other for carrying it forward. We may take another illustration. Suppose that H represents oatmeal, and that its use is abandoned for a more varied regimen, consisting of say oatmeal itself in less quantity, wheaten meal and maize-meal. We should have in cases like this also to get out new units of usage, and two aggregate-costs for the date of change, one based upon the original regimen ; one based upon the substituted regimen. In this way we ensure a high degree of real continuity in the succession of price-indexes.

PART IV.

THE SIGNIFICANCE OF PRICE-INDEXES AND CONCLUSIONS.

SYNOPSIS.

1. Further observations upon the continuity of price-indexes.
 2. The combination of price-indexes for various groups.
 3. The illusion of weighted price-indexes.
 4. The aggregate-expenditure or aggregate-cost method is alone valid.
 5. Application of price-indexes to questions of cost-of-living.
 6. True and unweighted average prices and their influence upon price-indexes.
 7. Consequences of error of applying unweighted means of prices.
 8. Common errors in regard to price-indexes.
 9. Price-indexes and cost of living in abnormal times.
 10. Conclusions.
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1. **Further observations upon the continuity of price-indexes.**—"Is there any sense in which a long series of price-indexes can be said to be rigorously continuous?" This is an important question, because during the last few decades the usage of commodities has been changing rapidly. We must, therefore, be prepared to make any future scheme for deducing price-indexes continually conform to actual usage, and, therefore, change with it. The mode of doing this has already been indicated, and no better way can be deduced than that of gradually changing the items in the schedule together with their quantities, for on these the total cost is estimated. The principle underlying this procedure may be stated in the following terms:—

- (i.) The commodity-basis, upon which a price-index for any particular purpose is based, must represent (both in respect of the items enumerated, and the quantities assigned to them) the usage for a given unit of time by the "average individual" of the particular class concerned. Thus if it be for the whole community it would include all commodities and the average usage of the whole population.
- (ii.) In questions where the standard of the commodity used (standard-of-living) is immaterial, what may be called "equivalent commodities" can be substituted for those which they must replace, despite the fact that the grade or quality is changed and that the price has been varied in consequence.

But this substitution cannot be effected if in the question to be answered, the element of changing quality or grade is material. In other words, the indications based upon price-indexes must always be interpreted with the actual facts under review, and in relation to them. Strictly they will apply only to these.

The question naturally suggests itself, "Why can we not ascertain the *price-indexes for classes of commodities*, and by properly weighting them obtain their weighted mean, and in this way get true continuity for the grand aggregate which they make up?" Moreover, "Would not the continuity of the price-indexes so ascertained be perfect?" is also a question which suggests itself. What has preceded, however, shews clearly that it would not; that in whatsoever manner we proceed, the results, apparently continuous, are, if we have a change in composite-unit constituting the basis of the comparisons, really discontinuous. It is, of course, obvious that a continuity, sufficient for practical purposes, can be had so long as the composite-unit is only slightly changed. But, as already explained, it cannot be too distinctly understood that the significance of price-indexes fades away as, with the lapse of time, the composite-unit changes. There is a sense, however, in which continuity could rigorously exist. Suppose, for example, that all commodities

increased or diminished in price at a uniform ratio in equal intervals of time, and that this was true not only of disappearing commodities, but also of new commodities entering into usage. This would be a case of rigorous continuity, although the composite-unit, on which the computation was based, might have been changing the whole time. It is equally obvious that as this condition is approached a continuity is implied, the rigorousness of which depends upon the degree of approach. We shall shew in the next section that we may combine different price-indexes so as to get a single one covering the combined groups. In both cases the continuity is perfectly rigorous only if the composite-unit, on which the results are based, remains unchanged; and it is only approximate, if there be any change therein. Nevertheless, its defect in approximation will be of small significance *if the change in the composite-unit is insignificant.*

2. The combination of price-indexes for various groups.—Let us suppose that price-indexes have been determined for independent groups of commodities, that is groups in which the same commodity does not reappear, as, for example, in the case of food, clothing, housing, etc.; and that the question of confirming these results arises, so that we can obtain a price-index applicable to the whole. The only *perfectly satisfactory* method is to add the aggregate expenditures (computed on the proper relative bases) and from these find the ratios of these aggregates. If we can know the relative-expenditures on the *initial (or basic) date* we can obtain a rigorously accurate result by using these as weights. So even if we know approximately the relative expenditure on any date we can deduce the price-index over all. For the rigorous result we must have the relative expenditure for the date which we make the basis as we shall shew.

In Australia for the years 1914 and 1917 the aggregates of expenditures on the composite-units (i.) groceries, (ii.) dairy products, (iii.) meat, and (iv.) house-rents, were as follow:—

Composite-unit.	Aggregates of Expenditure.		Ratios of Expenditure to Total.		Price-indexes.	
	1914.	1917.	1914.	1917.	1914=1.	1917=1.
Groceries ..	56588	67509	.24155	.24611	1.19299	0.83823
Dairy products ..	37688	44540	.16087	.16238	1.18180	0.84616
Meat ..	41919	64311	.17893	.23445	1.53417	0.65182
House-rents ..	98078	97943	.41865	.35706	0.99862	1.00138
Grand aggregate	234273	274303	1.00000	1.00000	1.17086	0.85407

The several price-indexes are the aggregate expenditure of 1917 divided by those of 1914. Now if we *weight* these by the relative expenditures in 1914, we get 1.17086, that is—

$$1.19299 \times .24155 + 1.18180 \times .16087 + 1.53417 \times .17893 + 0.99862 \times .41865 = 1.17086.$$

Similarly if we make 1917 the basic year, and make the weights 0.24611, etc. (i.e., the relative expenditures in the 1917 column) we get, as we should, the reciprocal of 1.17086, viz., 0.85407; that is—

$$.83823 \times .24611 + .84616 \times .16238 + .65182 \times .23445 + 1.00138 \times .35706 = .85407$$

The reason of this is readily demonstrated. Let the several aggregate expenditures be denoted by G, D, M, H and T ; thus $T = G + D + M + H$, with suffixes (say 4 and 7) to denote that they belong to particular years (1914 or 1917). Thus we have for the price-index over all for 1917, with 1914 as the basic year:—

$$(19) \dots \frac{G_7}{G_4} \cdot \frac{G_4}{T_4} + \frac{D_7}{D_4} \cdot \frac{D_4}{T_4} + \frac{M_7}{M_4} \cdot \frac{M_4}{T_4} + \frac{H_7}{H_4} \cdot \frac{H_4}{T_4} = \frac{G_7 + D_7 + M_7 + H_7}{T_4} = \frac{T_7}{T_4}.$$

Similarly, for the price-index over all for 1914, with 1917 as the basic year, we have

$$(20) \dots \frac{G_4}{G_7} \cdot \frac{G_7}{T_7} + \frac{D_4}{D_7} \cdot \frac{D_7}{T_7} + \frac{M_4}{M_7} \cdot \frac{M_7}{T_7} + \frac{H_4}{H_7} \cdot \frac{H_7}{T_7} = \frac{G_4 + D_4 + M_4 + H_4}{T_7} = \frac{T_4}{T_7};$$

The only other method that can lay claim to precision is the method based on geometric means of price-ratios weighted (as powers) with the mean relative expenditures of any compared dates. This method approximates very closely to the method of comparing the cost of the composite-unit at two different dates.

5. **Application of price-indexes to questions of cost of living.**—The cost of living is, of course, a flexible, not a fixed quantity. It depends upon several factors, for example :—

- (i.) The general purchasing-power of money.
- (ii.) The available margin between income and the cost of necessities of life.
- (iii.) Skill (a) in modifying one's regimen in order to deal with fluctuations in the prices of particular commodities to the best advantage, and (b) in the substituting of one commodity for another.
- (iv.) Economic adaptability, e.g., thrift, suitable selection of foods, etc.

From (ii.) it is evident that when there is *no margin*, and prices rise, (iii.) and (iv.) above are necessarily most in evidence.

A very large proportion of any population modifies its regimen according to price, and the season of those commodities, which fluctuate greatly, in available quantities, price, e.g., in respect of fruit, vegetables, eggs, game, and so on, buying less, or none at all, when prices are high, laying in supplies when they are low, etc.

Owing to this, the average use of food-commodities of a number of persons exhibits fairly well-defined seasonal fluctuations, these fluctuations being most strongly marked in the food-regimens of the most intelligent, thrifty and careful, and least marked on the whole, in the regimen of those to whom thrift is, from any cause, virtually of small moment. For this reason, if a constant regimen or composite-unit be adopted as a basis for measuring the purchasing-power of money in the case of foods, it might be urged that it does not represent actual usage at particular parts of the year; and consequently price-indexes based thereon are only hypothetically correct; * they do not correspond to actual facts. In a measure this is true, as regards cost of living, and most true in the case of those whose household economies are most intelligently directed to securing the fullest possible advantages of fluctuations in the prices of food-commodities. Nevertheless, if the usage of commodities is the average for the particular population (or class within the population) the error in using the constant regime throughout is small in the average result for a year. We shall now shew that even then a slight error exists.

6. True and unweighted average prices and their influence upon price-indexes.—

When we have ascertained, for any unit of time (say 1 year) embracing all fluctuations of price, the actual usage, and apply thereto the *average-price*, computed by allowing equal weight to the price, ascertained at equal intervals throughout the year (say weekly or monthly) we do *not* get the true expenditure *unless the usage is constant* throughout the year. For if the quantities of commodity A are a_1, a_2 , etc., to n terms, and the prices are p_1, p_2 , etc., the true *average price* (p_0) is the product of the prices by the quantities purchased divided by the total quantity, viz. :—

$$(22) \dots p_0 = (a_1 p_1 + a_2 p_2 + \dots + a_n p_n) / (a_1 + a_2 + \dots + a_n)$$

and is *not* merely the mean of the prices (p'_0), viz.,

$$(23) \dots p'_0 = (p_1 + p_2 + \dots + p_n) / n$$

unless, that is, the quantities purchased (a_1, a_2 , etc.) are all equal, in which case $p_0 = p'_0$. Thus we are not rigorously exact in applying the average-price as ordinarily ascertained, viz., the mean of the prices taken at equal small intervals of time, unless the usage is constant. It is, of course, quite *impracticable to apply the correction for variable usage for a large series of commodities*.

7. **Consequences of error of applying unweighted-means of prices.**—Practically the differences between weighted and unweighted *means of prices* are not seriously large, though in individual cases they may attain to the total difference between the lowest price and the true average, as for example, if a person bought a

* Of course if the usage is maintained constant they are absolutely correct.

year's supply of eggs, or of fruits, etc., at the lowest price, and preserved them. The error of assuming uniform usage of a commodity, that is, of supposing that the relative quantity based upon a year's total, may probably have applied thereto the unweighted-mean price is merely the difference between the weighted and unweighted means for the commodity. It gives an *appreciable advantage* only to those who are sufficiently watchful to take advantage of the periods of low prices, and whose forethought and circumstances enable them to lay in supplies. Consider an extreme where a commodity cost say 2/9 for 1 month, and 9d. for the rest of the year,* and let us suppose the usage to be as follows :—

10 persons use 1 per month for 11 months only	= 110, costing		990d.
1 person uses 1 per month for 12 months	= 12, costing		132d.
11 persons use $\frac{1}{11}$ per month for 12 months	= 122, costing		1122d.

The true average cost is, therefore, $9\frac{1}{11}$ pence. The unweighted average of the prices is $(11 \times 9 + 1 \times 33) \div 12 = 11$ pence, which applied to the total represents $11 \times 12 \times 11d. = 1452d.$, which is in excess 330d. or about 13 per cent. Suppose that expenditure on this commodity constituted 2 per cent. of the total expenditure,† the effect would be to cause an error of only 0.26 per cent. in the price-index. Such an error, though not exactly an insensible one, is practically negligible, and in all actual cases errors of this kind would be much smaller.

8. **Common errors in regard to price-indexes.**—It is obvious, and it was shewn in Part II, section 11., that the cost of two composite-units somewhat of the same general character must often differ sensibly. This has frequently given rise to an impression that the price-indexes may be greatly prejudiced by this fact. This is an illusion arising from the failure to recognise that the result is of a differential character. The percentage of change in the aggregate-cost of a composite-unit is *not* the measure of change in the price-index. Thus suppose a change increases the aggregate cost 5 per cent. on one occasion only, and let us suppose that the increase of the price-index is 30 per cent. Then we should have price-index $130 \div 100 \times 1000 = 1300$; with a correction of 5 to both the 100 and 130 it is $135 \div 105 \times 1000 = 1286$; that is to say, the result has been affected only about 1 per cent. More generally, if one composite-unit gives expenditures *A* and *B* on two occasions, and a second composite-unit is about *m* times the former (in which *m* may have any value whatsoever), and if also minor differences of price, etc., cause differences *mh* and *mk* in the two, we shall then have for the price-indexes :—

$$A/B; \text{ or } (A + h) / (B + k), \text{ or } (mA + mh) / (mB + mk)$$

Thus on effecting the divisions we have :—

$$(24) \dots\dots \frac{A}{B}; \text{ or } \frac{mA + mh}{mB + mk} = \frac{A}{B} \left\{ 1 + \left(\frac{h}{A} - \frac{k}{B} \right) + \epsilon \right\};$$

in which ϵ is a quantity depending upon the powers of the very small quantities h/A and k/B . Since *h* and *k* are relatively small to *A* and *B*, the whole quantity between the braces is very nearly unity, and is in general negligible.

9. **Price-indexes and cost-of-living in abnormal times.**—War conditions, droughts, failures of crops, and other economic disturbances, while they do not *always* produce such a bouleversement as to vitiate all price-relations, and even make impossible supply of necessary commodities, *often* do so. In such an event the method of computing price-indexes in not nullified but the price-index loses temporarily (or it may be permanently) its significance, because the usage of the community must perforce be altered. The basic composite-unit no longer represents the actual usage of the community. For this reason no price-index has any valid general application in such a case. The practical solution of questions of cost-of-living in abnormal times may turn not upon price-indexes, but upon available quantities of commodities, their food-values, their prices *inter se*; or finally, in extreme cases (famine, devastation by war, etc.) the practical solution may be reduced to the very limited possibilities of the situation.

10. **Conclusions.**—The following conclusions are either directly indicated in the preceding examination of this question, or they are necessary consequences of what has been established :—

* Say eggs at 2s. 9d. per dozen, and later at 9d.

† Eggs represent about 1 to 2 per cent. of the expenditure for a whole year.

- (i.) The purchasing-power of money for any two localities or any two dates varies according to its specific purpose, that is according to usage in respect of the scope, character and quantities of the commodities used.
- (ii.) To accurately measure the purchasing-power appropriate for each such purpose, a composite-unit must be employed, which unit must consist of definite quantities of a specific series of commodities, and must, moreover, represent actual usage.
- (iii.) The ratio of the purchasing-power between any two localities or dates is the reciprocal, or inverse of the cost, of the appropriate composite-unit (either for the two localities or the two dates.)
- (iv.) Variations of purchasing-power are best shewn by means of price-indexes, which represent the relative cost in the second case (second date or locality) as compared with the first, that in the first or basic date (or locality) being denoted by 100, 1,000, or 10,000, etc., according to the degree of precision required.
- (v.) Price-indexes or ratios can be accurately combined by weighting them according to the relative expenditures on each, in the basic year only.
- (vi.) It is preferable, however, to combine the aggregates of expenditure directly.
- (vii.) Better, and of course more intelligible, results can be obtained by making the composite-unit include all essential commodities for the specific purpose of the index.
- (viii.) The applicability of price-indexes is strictly limited to the specific purpose, which constituted the guide in formulating the composite-units on which they were founded.
- (ix.) Price-indexes can be combined, if, among them all the weighting has been based upon the expenditure of the basic year, the relative quantities indicating the actual usage.
- (x.) The aggregate-cost of (or expenditure on) the composite-unit is not only the only accurate way of finding a price-index, it is also arithmetically the most simple.
- (xi.) In practical cases where the question of *standard* of living is affected, we must take care that, in general, the commodities are also accurately identifiable in respect of quality or grade.
- (xii.) Where these are with difficulty identifiable, it may be better to exclude the commodities, or to ascertain the effect on the price-index which their uncertainty introduces.
- (xiii.) Accurately computed price-indexes, from the cost of definite composite-units, though rigorously applicable only to the units on which they have been based, can be regarded as generally applicable to any case of like nature.
- (xiv.) They can also be regarded as applicable whenever there is no reason to suppose that the change of price of the unincluded commodities sensibly differs.
- (xv.) Whenever change in price affects quantitatively the usage of the several commodities, the only satisfactory basis of comparison is a composite-unit, which is the mean of the usage on the two occasions compared.
- (xvi.) The quantities in the composite-unit must be accurate relatively to each other, but are unaffected by any common multiple.
- (xvii.) Provided the composite-unit is comprehensive, meticulous accuracy in determining the quantities (or mass-units) of each commodity is unnecessary: they must of course be fairly accurate, however.
- (xviii.) Where a commodity changes in grade in such a manner that the old grade disappears and the new grade takes its place, the fact of variation of grade may for most purposes be ignored.
- (xix.) Price-indexes designed to indicate what change of wages is necessary in respect of commodities, should be based upon the average usage of the identifiable commodities.
- (xx.) Price-variations due to change of grade in commodities, or to changes in the commodities themselves, nullify comparisons, inasmuch as they introduce the effect of change of standard, (*e.g.*, standard of living).

- (xxi.) In dealing with price-indexes in relation to questions of a so-called living (or minimum) wage account should be taken of the lowest suitable quality of a commodity, *i.e.*, the commodities may be regarded as made up of two elements, *viz.*, the *necessary element* and the *luxury element*. The former is alone of moment.
- (xxii.) The cost of commodities must be based, not upon mere initial cost, but the cost per some definite unit of time, with all circumstances of usage taken into consideration. Thus the ratio of the initial cost of commodities used for a long time and subject to repair at a moderate cost does not measure their price-relationship.
- (xxiii.) Price-indexes deduced on the aggregate-expenditure method, depend, as is obvious, on the precision with which prices are ascertained.
- (xxiv.) Thus prices, in order to give results of the highest precision, should be those which constitute a *true average*.
- (xxv.) An unweighted average of prices will nevertheless give results which are sensibly correct.
- (xxvi.) Price-indexes are fully comparable for any period during which the composite-unit on which they are based not only remains unchanged but also substantially represents the specific usage (general or particular) of the commodity to which it is applied.
- (xxvii.) As the commodity-usage changes, the significance of a price-index changes *pari passu*, until finally the index-numbers (say for widely separated dates) are unrelated, *i.e.*, they have no significance in relation to each other.
- (xxviii.) Nevertheless, if for points of time not widely separated the composite-units are generally similar, they are significant for most purposes, and may be treated as applicable.
- (xxix.) Comparisons of the relationship between money and commodities for widely separated dates, when presumably the composite-units would be very dissimilar, *must be founded, not upon price-indexes, but upon other bases*, since in such cases no common basis exists for the measurement of the purchasing-power of money in relation to commodities.
- (xxx.) Although for widely separated dates comparisons can be made between the unit of money, and (a) the average cost of living, (b) the most frequent expenditure on living, (c) the food-values purchaseable with such a unit, and so on, such measurements give results which have no definite and determinable relation to price-indexes.
- (xxxi.) In abnormal times, price-indexes cease to have any general significance in the degree the composite-unit, on which they are based, ceases to represent the actual usage of the commodities.
- (xxxii.) Any attempt to apply price-indexes to questions concerning the cost of living, must take cognisance of the normality or otherwise of the general conditions.
- (xxxiii.) Abnormal times involve the consideration of questions of cost of living upon special bases (*e.g.*, the possibilities of obtaining commodities; the compulsion to change because of extraordinary prices; the possibility of variations of food to secure adequate food-values, and so on.)
- (xxxiv.) Attempts to vary the composite-unit for fractions of a year so as to include insignificant changes in the bases for determining price-indexes are impracticable, and make only insensible differences in applications of price-indexes to questions of cost of living.
- (xxxv.) So long as the regimen adopted represents approximately the general usage, price-indexes, computed by the aggregate-expenditure method, may be legitimately employed to determine the equivalent wages payment necessary to maintain the same commodity purchasing-value for different periods.
- (xxxvi.) The fact that some item or items of expenditure, not included in the regimen adopted, may be shewn to have increased or decreased to a greater extent than that indicated by the price-index does not necessarily vitiate the applicability of the index-number for the purpose of equating wages.
- (xxxvii.) It is only when the whole of that part of the expenditure, not included in the regimen, has varied differently from the price-index, that the modification of the price-index can be justified.

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